

USING THE ANALOG DEVICES ACTIVE FILTER DESIGN TOOL

INTRODUCTION

The Analog Devices Active Filter Design Tool is designed to aid the engineer in designing all-pole active filters.

The filter design process is a two part effort. First, the response of the filter is determined. By this it is meant that the attenuation and/or phase response of the filter is defined. In step two the topology of the filter, how it is built, is defined. This application note is intended to help in step one. Several different standard responses are discussed, and their attenuation, group delay, step response and impulse response are presented. The filter tool is then employed to design the filter.

An example will be worked out.

STANDARD RESPONSES

There are many transfer functions that may satisfy the attenuation and/or phase requirements of a particular filter. The one that you choose will depend on the particular system. The importance of the frequency domain response vs. the time domain response must be determined. Also, both of these might be traded off against filter complexity, and thereby cost.

BUTTERWORTH

The Butterworth filter is the best compromise between attenuation and phase response. It has no ripple in the passband or the stopband and because of this is sometimes called a maximally flat filter. The Butterworth filter achieves its flatness at the expense of a relatively wide transition region from passband to stopband, with average transient characteristics.

The values of the elements of the Butterworth filter are more practical and less critical than many other filter types.

The frequency response, group delay, impulse response and step response are shown in fig. 1. The pole locations and corresponding ω_0 and α terms are tabulated in fig. 12.

CHEBYSHEV

The Chebyshev (or Chevyshev, Tschebychev, Tschebyscheff or Tchevysheff, depending on how you translate from Russian) filter has a smaller transition region than the same-order Butterworth filter, at the expense of ripples in its passband. This filter gets its name from the Chebyshev criterion, which minimizes the height of the maximum ripple.

Chebyshev filters have 0 dB relative attenuation at DC. Odd order filters have an attenuation band that extends from 0 dB to the ripple value. Even order filters have a gain equal to the passband ripple. The number of cycles of ripple in the passband is equal to the order of the filter.

The Chebyshev filters are typically normalized so that the edge of the ripple band is at $\omega_0 = 1$.

The 3 dB bandwidth is given by:

$$A_{3dB} = \frac{1}{n} \cosh^{-1} \left(\frac{1}{\varepsilon} \right) \quad (1)$$

This is tabulated in Table 1.

The frequency response, group delay, impulse response and step response for the various Chebyshev filters are shown in figs. 2 - 6. The pole locations and corresponding ω_0 and α terms are tabulated in figs. 13-17.

BESSEL

Butterworth filters have fairly good amplitude and transient behavior. The Chebyshev filters improve on the amplitude response at the expense of transient behavior. The Bessel filter is optimized to obtain better transient response due to a linear phase (i.e. constant delay) in the passband. This means that there will be relatively poorer frequency response (less amplitude discrimination).

The frequency response, group delay, impulse response and step response for the Bessel filter are shown in fig. 7. The pole locations and corresponding ω_0 and

α terms are tabulated in fig. 18.

LINEAR PHASE WITH EQUIRIPPLE ERROR

The linear phase filter offers linear phase response in the passband, over a wider range than the Bessel, and superior attenuation far from cutoff. This is accomplished by letting the phase response have ripples, similar to the amplitude ripples of the Chebyshev. As the ripple is increased, the region of constant delay extends further into the stopband. This will also cause the group delay to develop ripples, since it is the derivative of the phase response. The step response will show slightly more overshoot than the Bessel and the impulse response will show a bit more ringing.

The frequency response, group delay, impulse response and step response for equiripple filters with error of $.05^\circ$ and $.5^\circ$ are shown in figs. 8 & 9 respectively. The pole locations and corresponding ω_0 and α terms are tabulated in figs. 19 & 20.

GUASSIAN TO 6 dB & GUASSIAN TO 12 dB

Gaussian to 6 & 12 dB are transitional filters, which are a compromise between

ORDER	.01dB	.1dB	.25dB	.5dB	1dB
2	3.30362	1.93432	1.59814	1.38974	1.21763
3	1.87718	1.38899	1.25289	1.16749	1.09487
4	1.46690	1.21310	1.13977	1.09310	1.05300
5	1.29122	1.13472	1.08872	1.05926	1.03381
6	1.19941	1.09293	1.06134	1.04103	1.02344
7	1.14527	1.06800	1.04495	1.03009	1.01721
8	1.11061	1.05193	1.03435	1.02301	1.01316
9	1.08706	1.04095	1.02711	1.01817	1.01040
10	1.07033	1.03313	1.02194	1.01471	1.00842

Table 1: Chebyshev Cutoff Frequency To -3 dB Frequency

a Gaussian filter, which is similar to a Bessel, and the Chebyshev. A transitional filter has nearly linear phase shift and smooth, monotonic rolloff in the passband. Above the passband there is a break point beyond which the attenuation increases dramatically compared to the Bessel, and especially at higher values of n .

The Gaussian to 6 dB filter has better transient response than the Butterworth in the passband. Beyond the breakpoint, which occurs at $\omega = 1.5$, the rolloff is similar to the Butterworth.

The Gaussian to 12 dB filter's transient response is much better than Butterworth in the passband. Beyond the 12 dB breakpoint, which occurs at $\omega = 2$, the attenuation is less than Butterworth.

The frequency response, group delay, impulse response and step response for Gaussian to 6 & 12 dB are shown in figs. 10 & 11 respectively. The pole locations and corresponding ω_0 and α terms are tabulated in figs. 21 & 22..

USING THE PROTOTYPE RESPONSE CURVES

The response curves and the design tables for several of the lowpass prototypes of the all pole responses discussed earlier in the app note are now cataloged. All the curves are normalized to a -3 dB cutoff frequency of 1 Hz. This allows direct comparison of the various responses. In all cases the amplitude response for the 2 through 10 pole cases for the frequency range of .1 Hz. to 10 Hz. will be shown. Then a detail of the .1 Hz to 2 Hz. passband will be shown. The group delay from .1 Hz.

to 10 Hz., the impulse response and step response from 0 sec. to 5 sec. will also be shown.

To use these curves to determine the response of real life filters, they must be denormalized. In the case of the amplitude responses, this is simply accomplished by multiplying the frequency axis by the desired cutoff frequency F_C . To denormalize the group delay curves we divide the delay axis by $2\pi F_C$, and multiply the frequency axis by F_C , as before. Denormalize the step response by dividing the time axis by $2\pi F_C$. Denormalize the impulse response by dividing the time axis by $2\pi F_C$ and multiplying the amplitude axis by the same amount.

For a highpass filter, simply invert the frequency axis for the amplitude response. In transforming a lowpass filter into a highpass filter the transient behavior is not preserved. Zverev provides a computational method for calculating these responses.

In transforming a lowpass into a narrowband bandpass, the 0 Hz. axis is moved to the center frequency F_0 . It stands to reason that the response of the bandpass case around the center frequency would then match the lowpass response around 0 Hz. The frequency response curve of a lowpass filter actually mirrors itself around 0 Hz., although we generally don't concern ourselves with negative frequency.

To denormalize the group delay curve for a bandpass filter divide the delay axis by πBW , where BW is the 3 dB bandwidth in Hz. Then multiply the frequency axis by $BW/2$. In general, the delay of the bandpass filter at F_0 will be

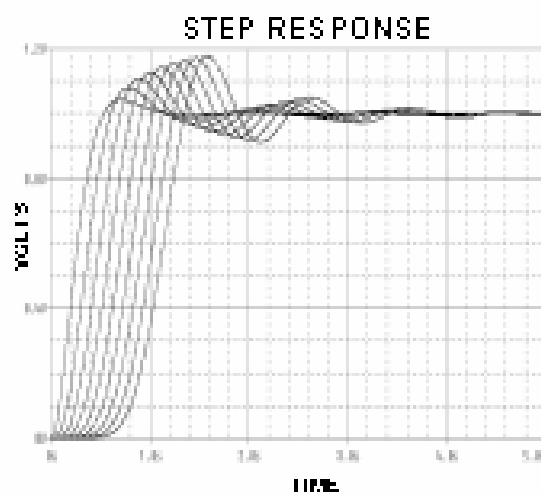
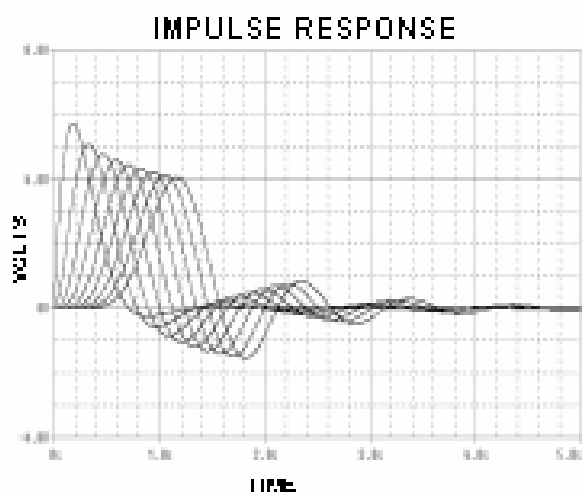
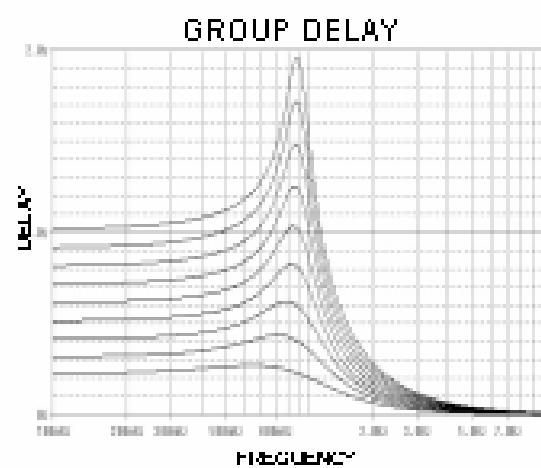
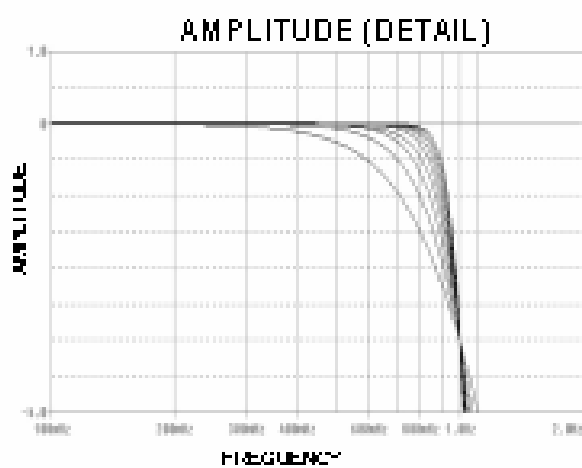
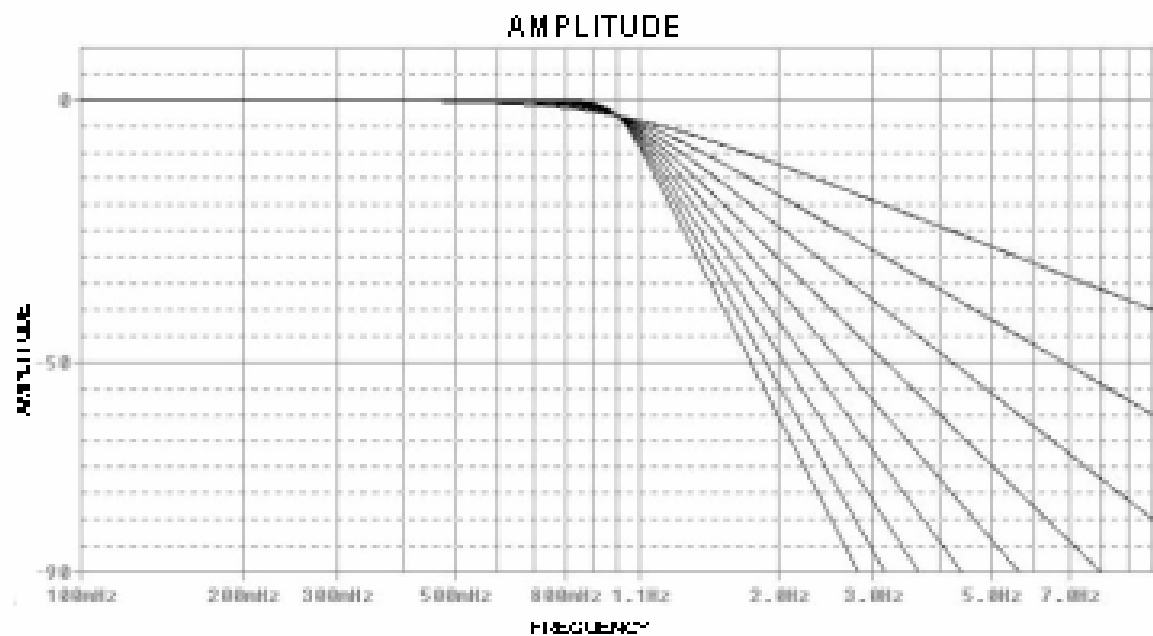
twice the delay of the lowpass prototype with the same bandwidth at 0 Hz. This is due to the fact that the lowpass to bandpass transformation results in a filter with order $2n$, even though it is typically referred to it as having the same order as the lowpass filter we derive it from. This approximation holds for narrowband filters. As the bandwidth of the filter is increased, some distortion of the curve occurs. The delay becomes less symmetrical, peaking below F_0 .

The envelope of the response of a bandpass filter resembles the step response of the lowpass prototype. More exactly, it is almost identical to the step response of a lowpass filter having half the bandwidth. To determine the envelope response of the bandpass filter, divide the time axis of the step response of the lowpass prototype by πBW , where BW is the 3 dB bandwidth. The previous discussions of overshoot, ringing, etc.

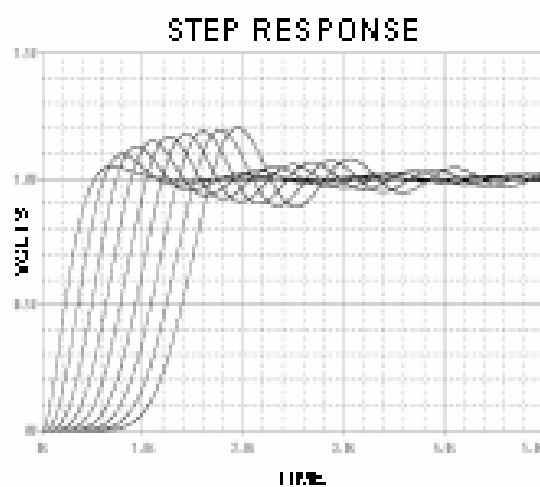
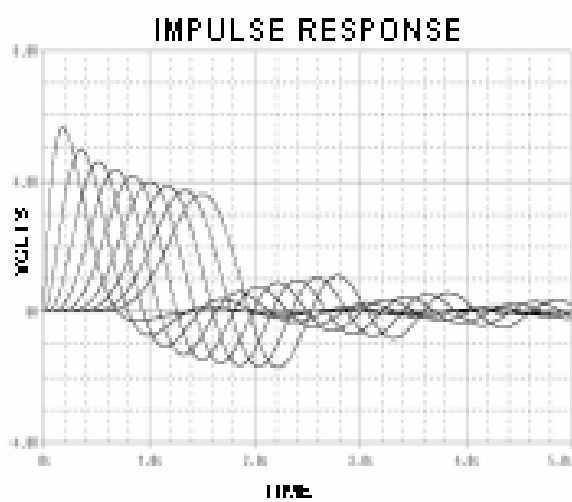
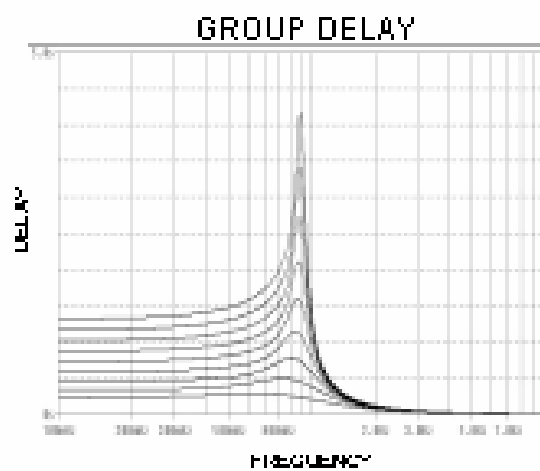
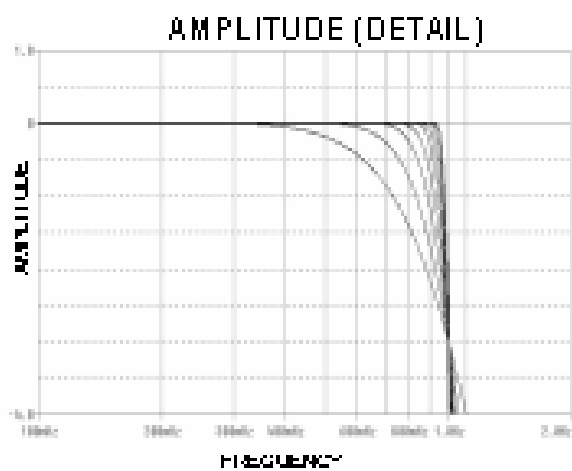
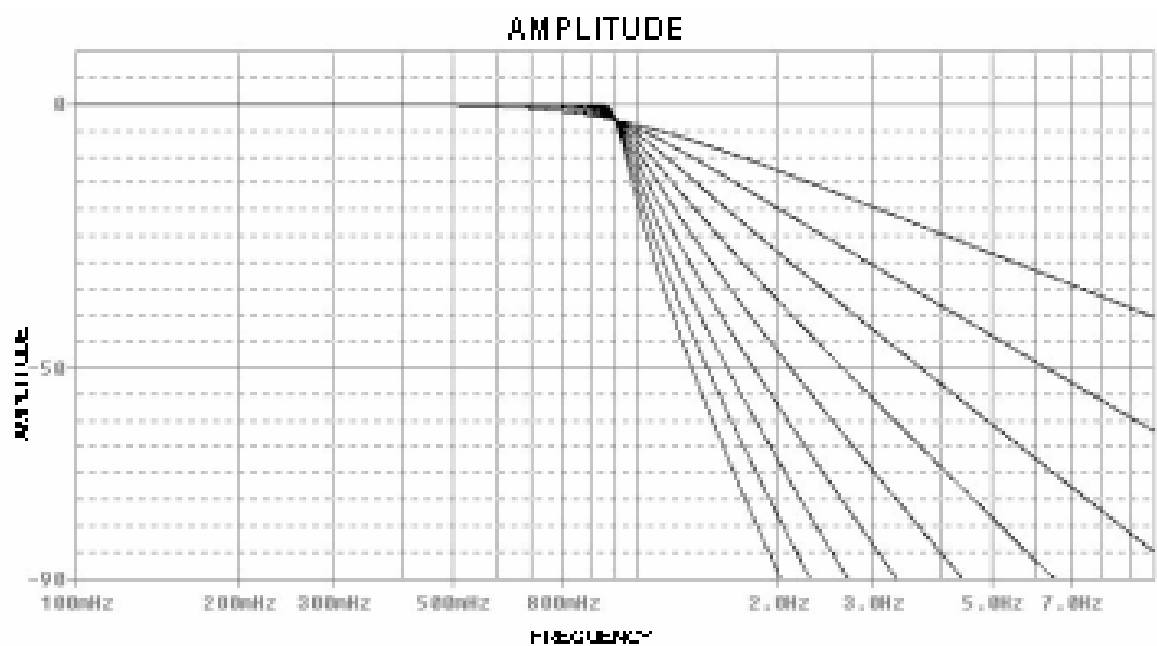
can now be applied to the carrier envelope.

The envelope of the response of a narrowband bandpass filter to a short burst (that is where the burst width is much less than the rise time of the denormalized step response of the bandpass filter) burst of carrier can be determined by denormalizing the impulse response of the low pass prototype. To do this multiply the amplitude axis and divide the time axis by πBW , where BW is the 3 dB bandwidth. It is assumed that the carrier frequency is high enough so that many cycles occur during the burst interval.

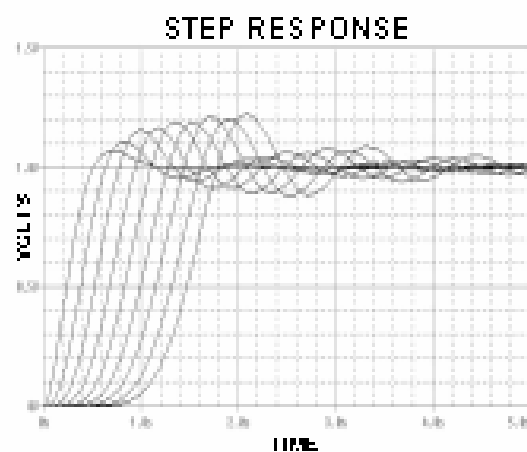
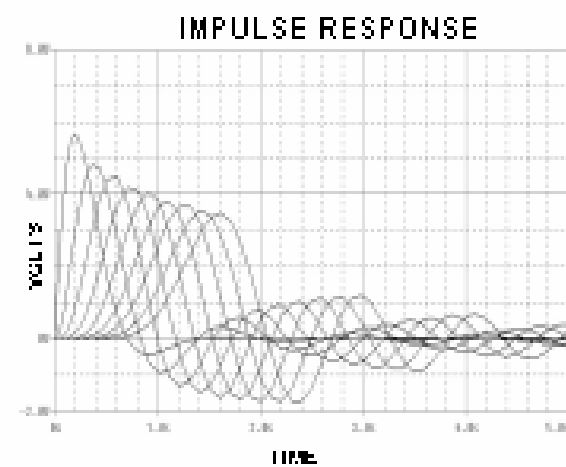
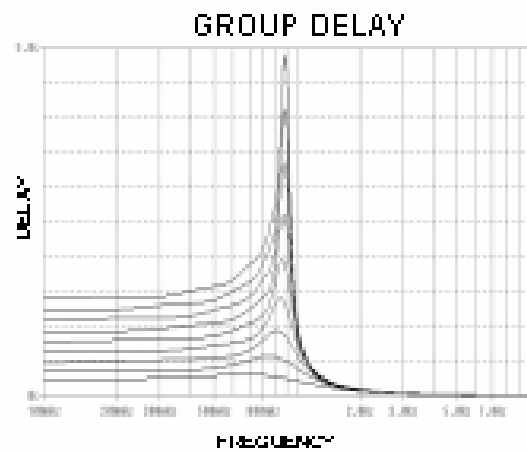
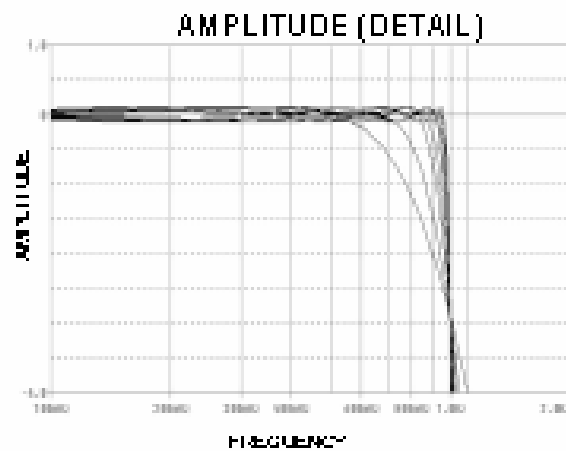
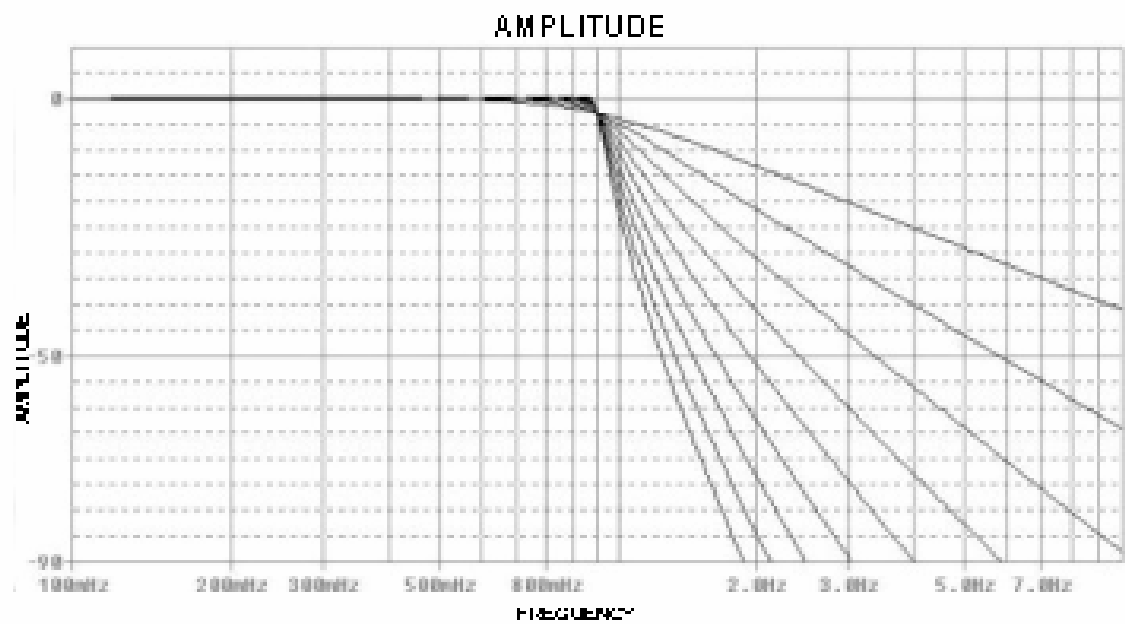
While the group delay, step and impulse curves cannot be used directly to predict the distortion to the waveform caused by the filter, they are a useful figure of merit when used to compare filters.



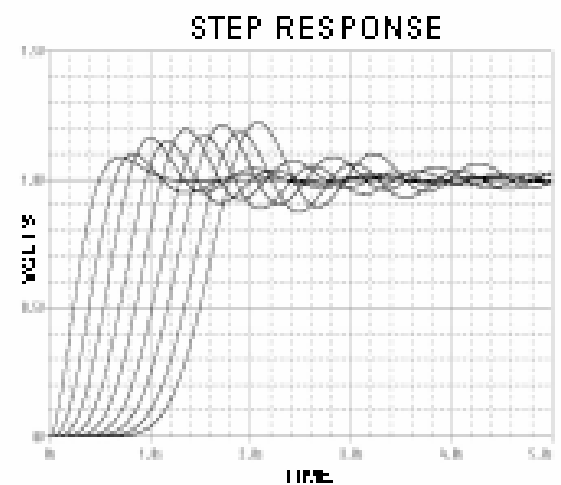
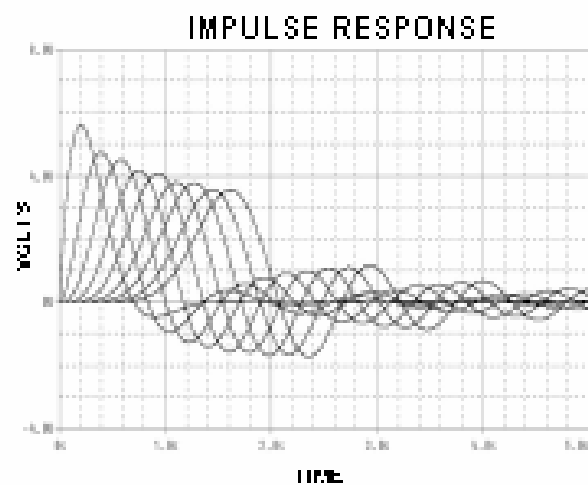
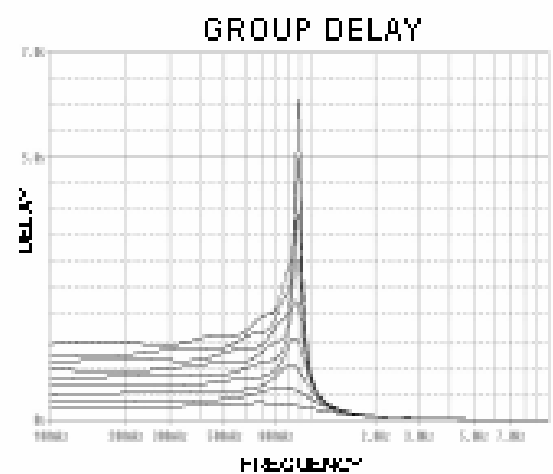
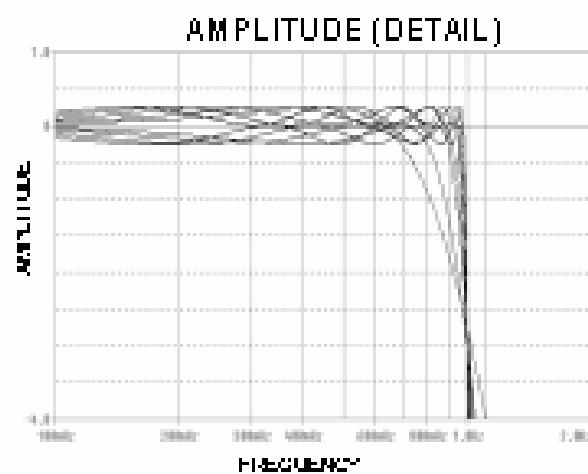
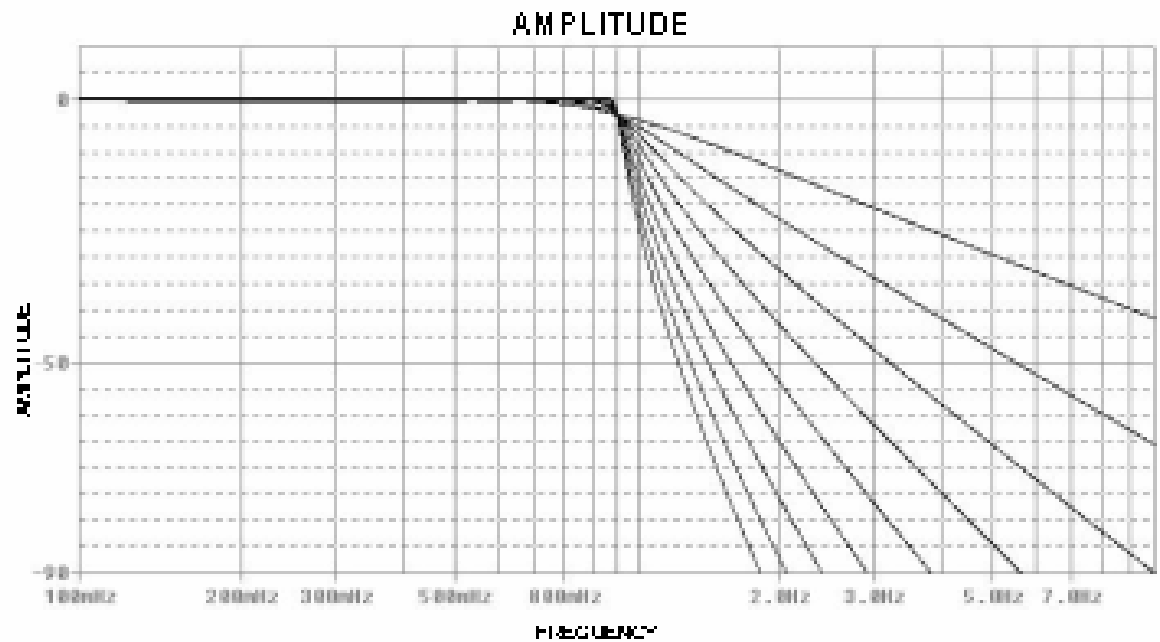
**Butterworth Response
Figure 1**



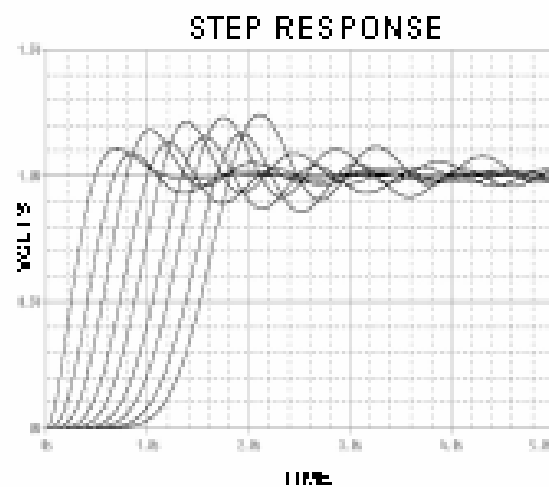
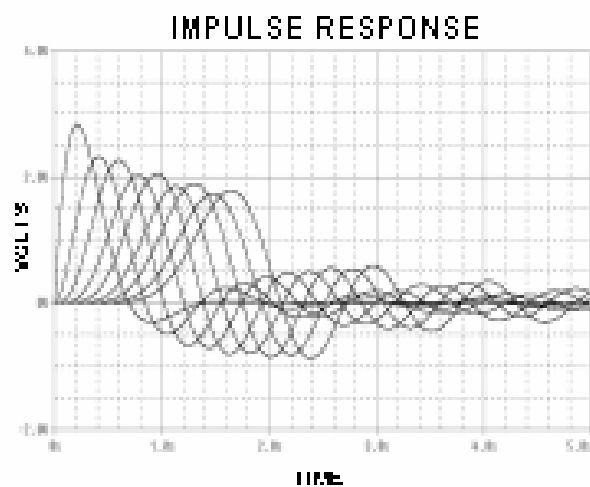
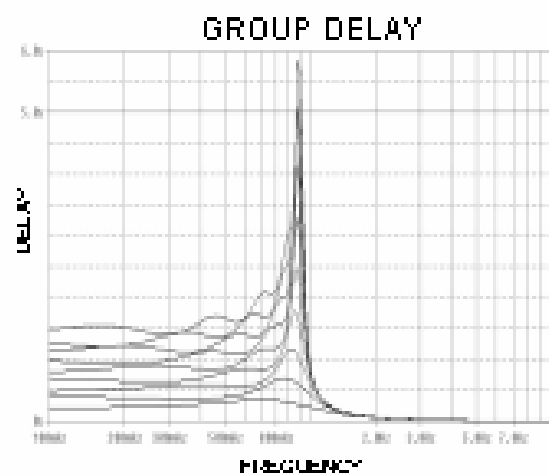
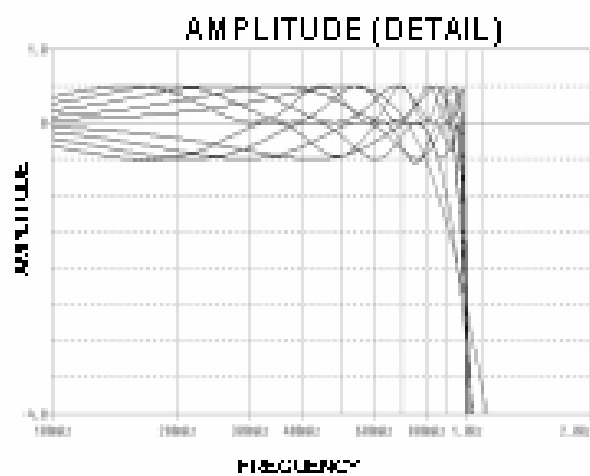
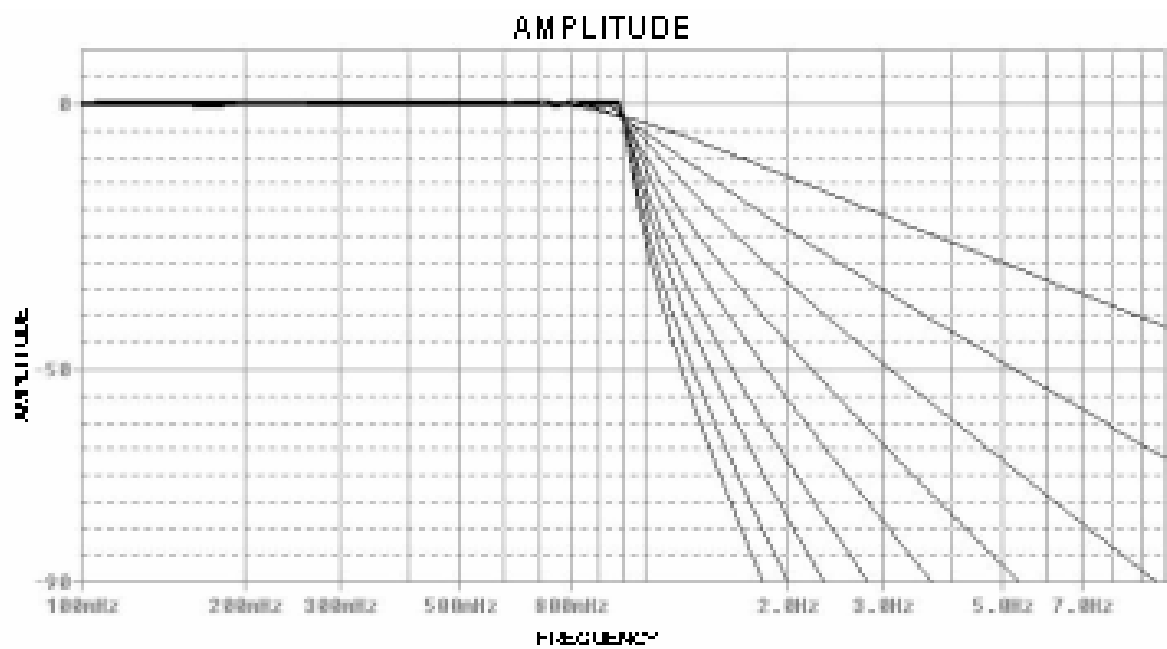
**.01 dB Chebyshev Response
Figure 2**



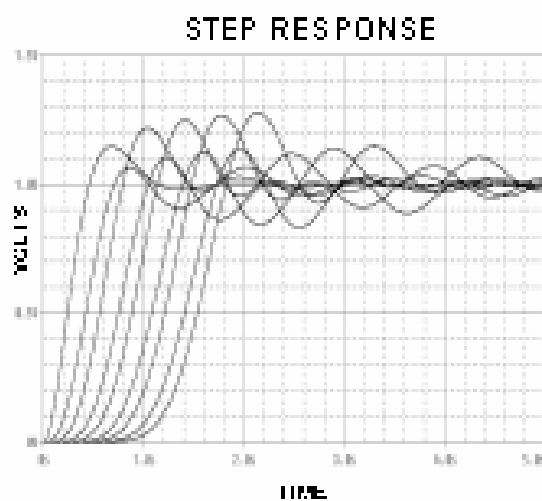
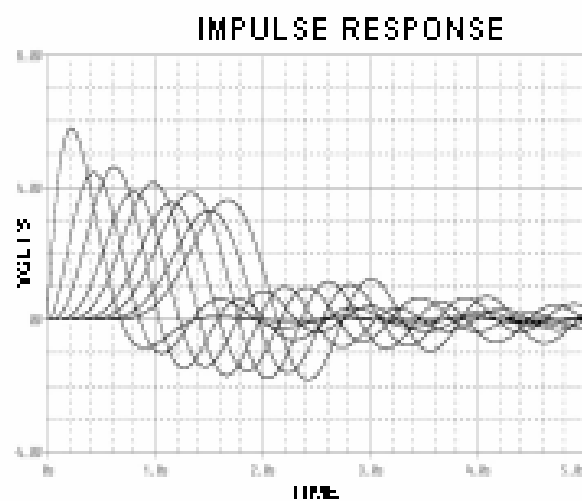
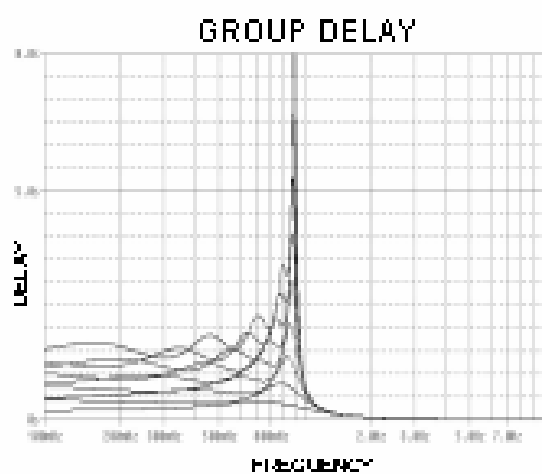
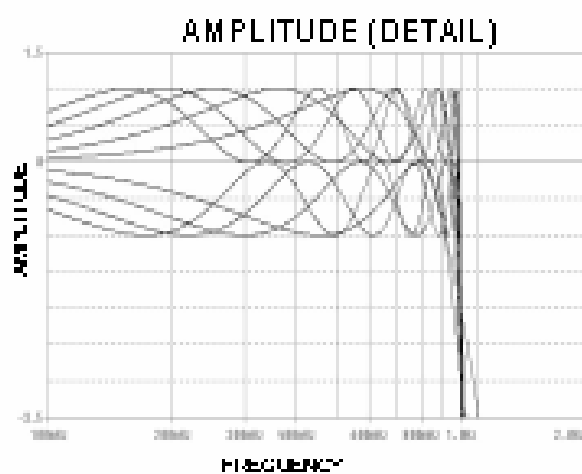
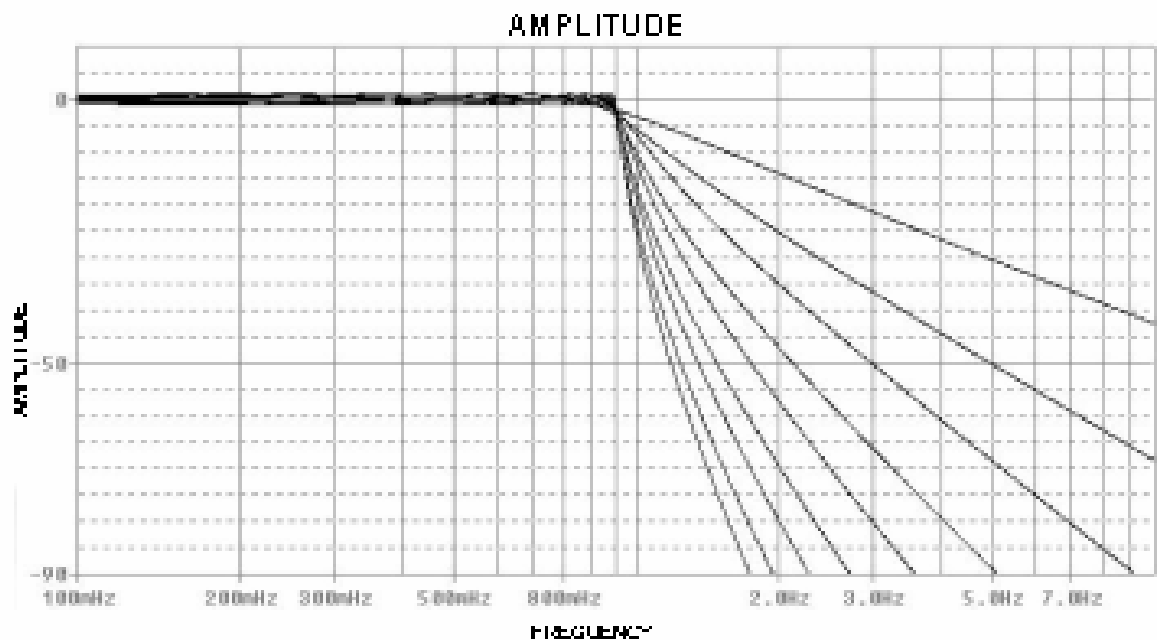
.1 dB Chebyshev Response
Figure 3



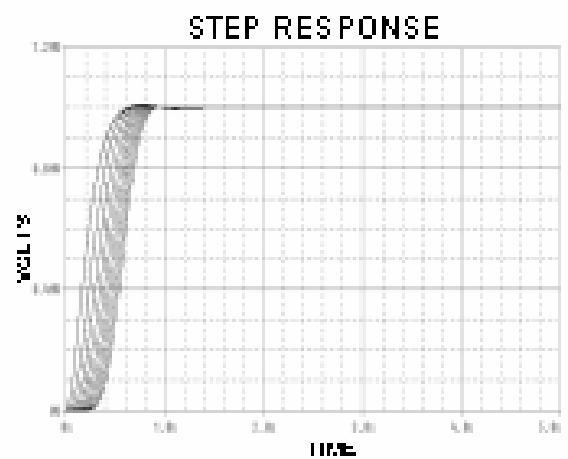
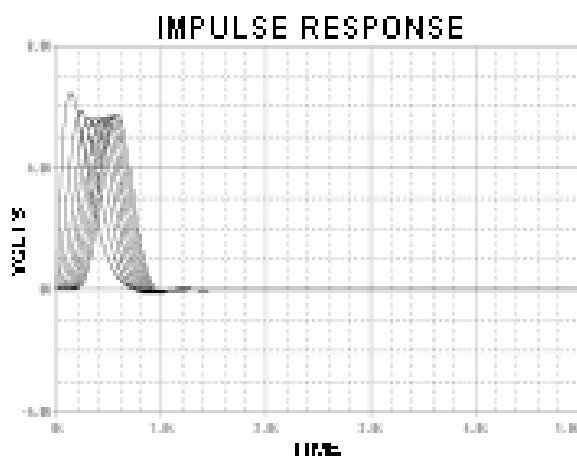
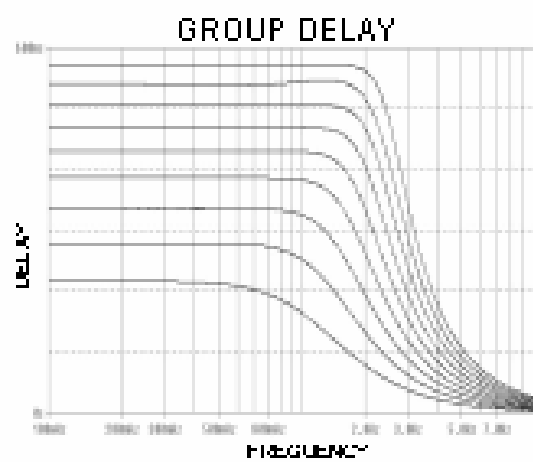
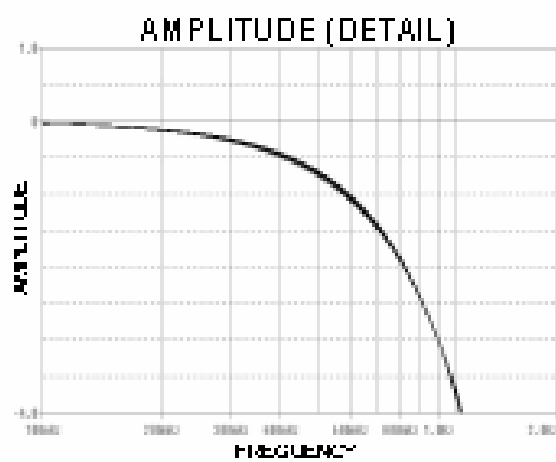
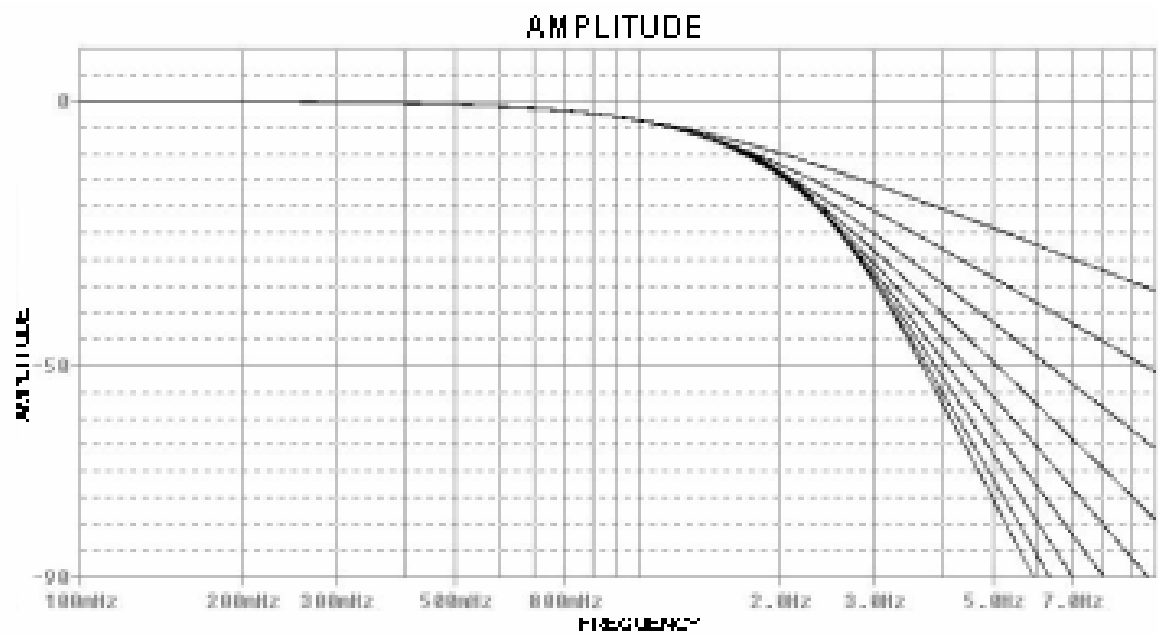
.25 dB Chebyshev Response
Figure 4



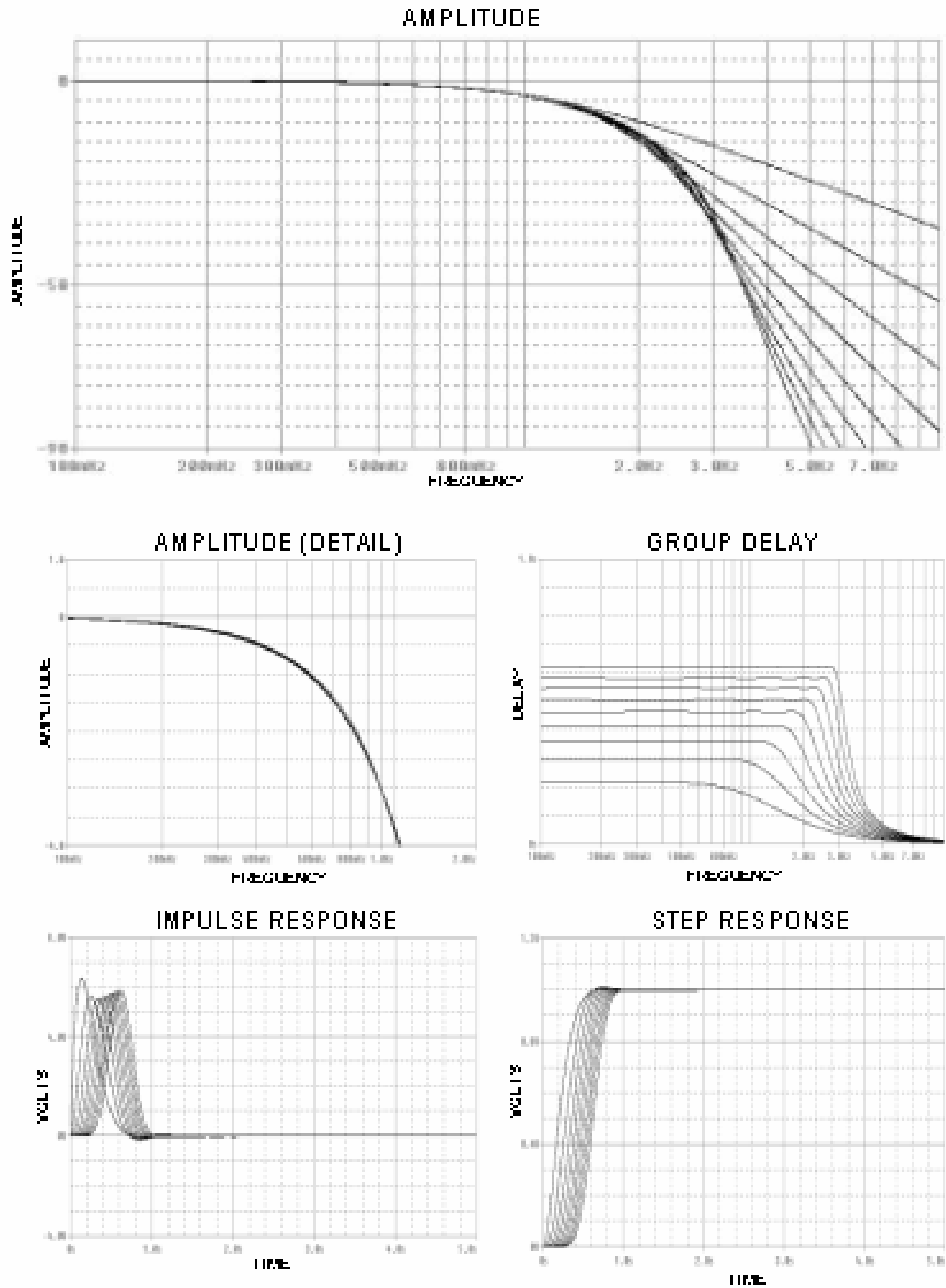
**.5 dB Chebyshev Response
Figure 5**



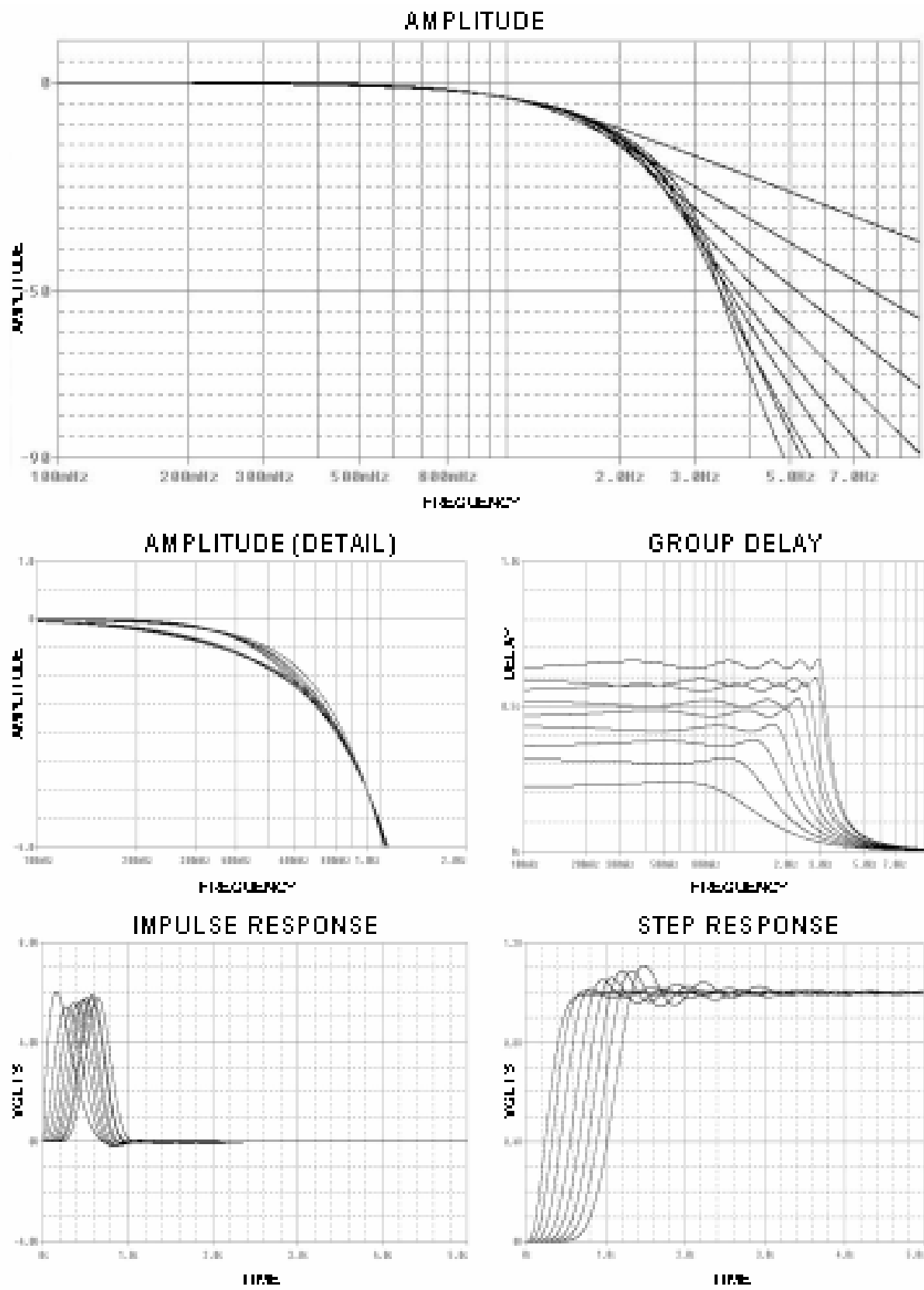
**1 dB Chebyshev Response
Figure 6**



**Bessel Response
Figure 7**



Linear Phase with Equiripple Error of $.05^\circ$ Response
Figure 8



Linear Phase with Equiripple Error of 5° Response
Figure 9

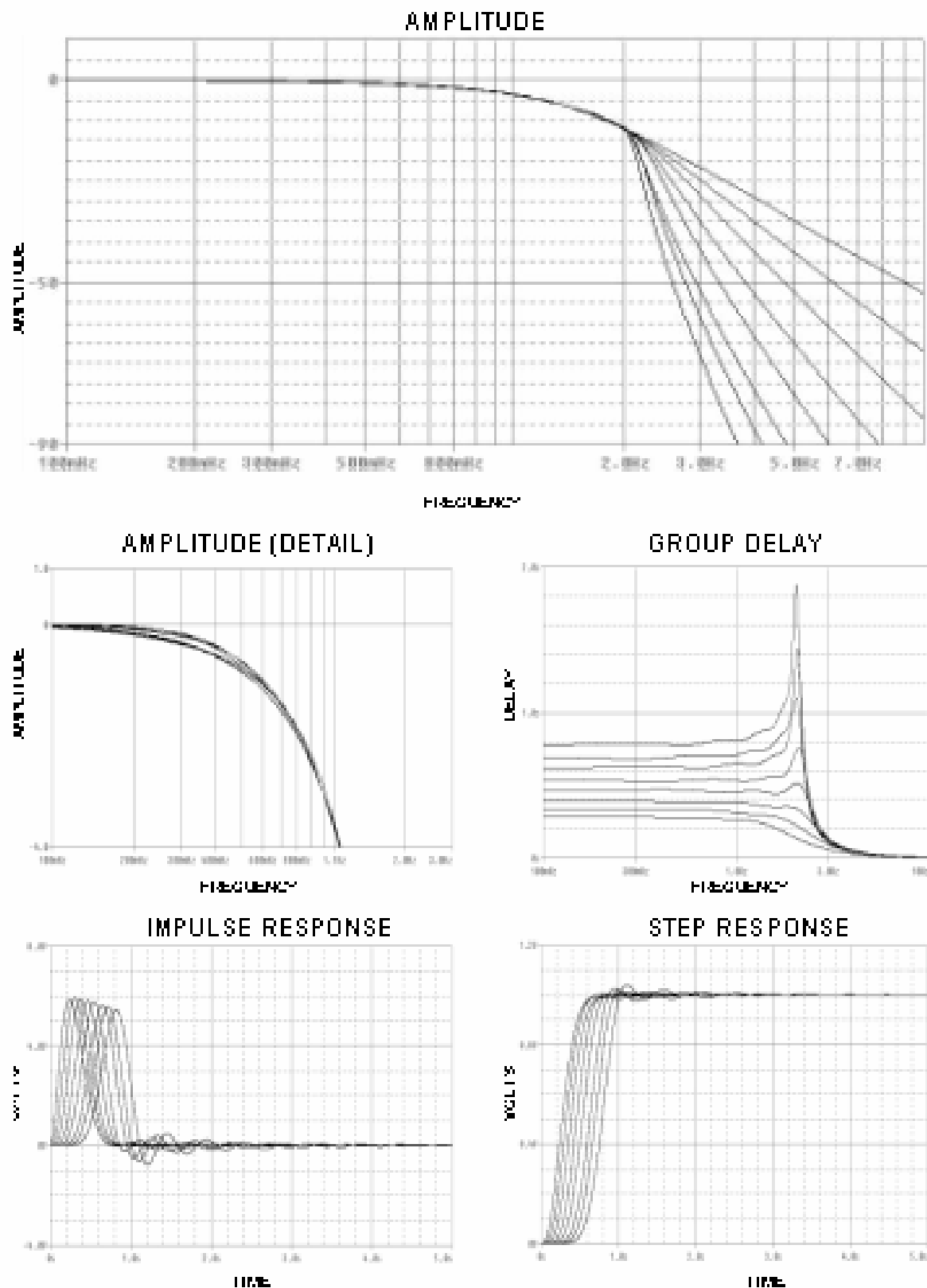
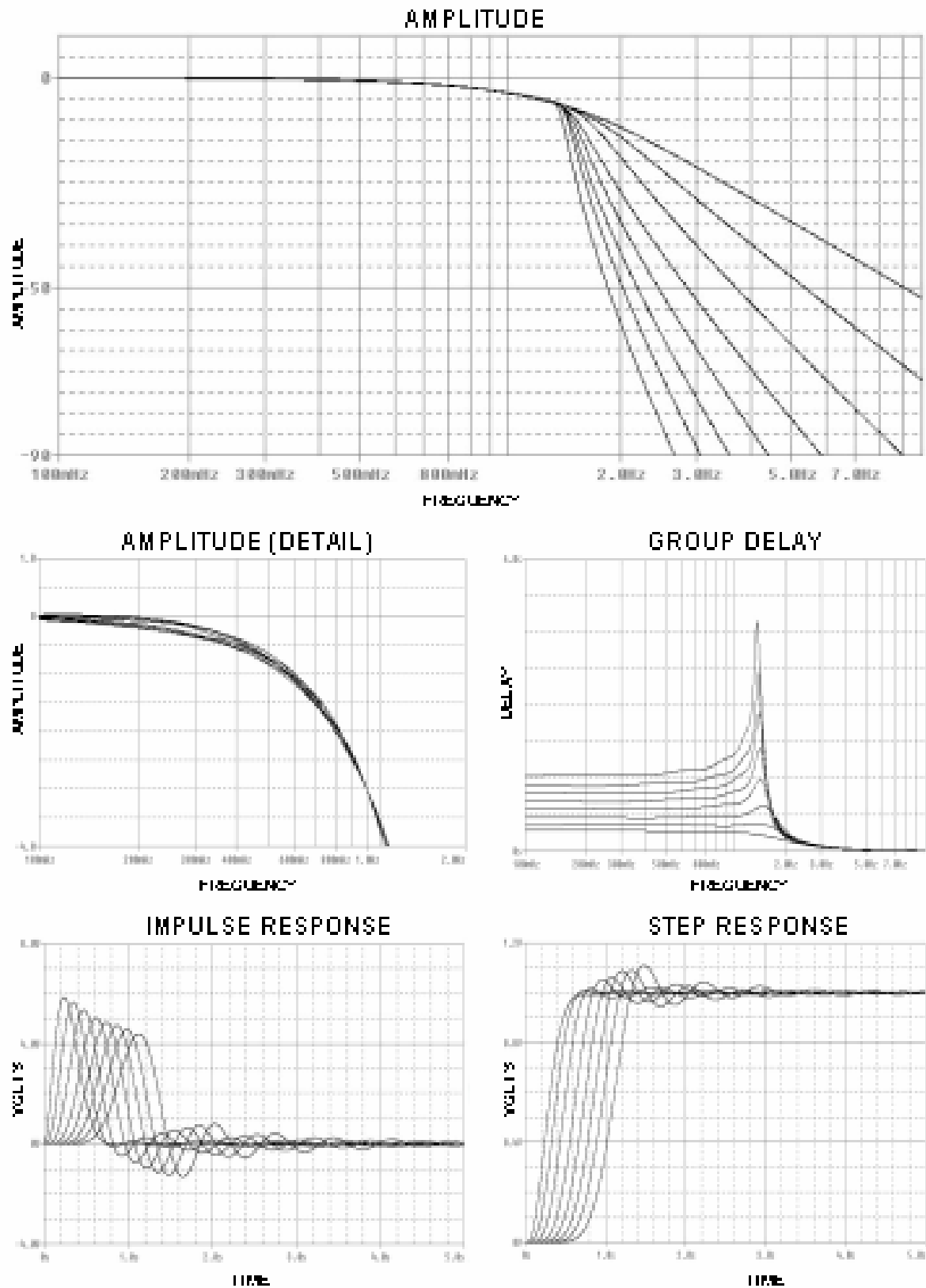


Figure 9 Gaussian to 12 dB Response
 Figure 10



Gaussian to 6 dB Response
Figure 11

Butterworth Design Table Fig.11

ORDER	SECTION	REAL PART	IMAGINARY PART	F _O	α	Q	-3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	0.6743	0.7075	0.9774	1.3798	0.7247		0.2142	0.0100
3	1	0.4233	0.8663	0.9642	0.8780	1.1389		0.7558	2.0595
	2	0.8467		0.8467			0.8467		
4	1	0.6762	0.3828	0.7770	1.7405	0.5746	0.6069		
	2	0.2801	0.9241	0.9656	0.5801	1.7237		0.8806	5.1110
5	1	0.5120	0.5879	0.7796	1.3135	0.7613		0.2889	0.0827
	2	0.1956	0.9512	0.9711	0.4028	2.4824		0.9309	8.0772
	3	0.6328		0.6328			0.6328		
6	1	0.5335	0.2588	0.5930	1.7995	0.5557	0.4425		
	2	0.3906	0.7072	0.8079	0.9670	1.0342		0.5895	1.4482
	3	0.1430	0.9660	0.9765	0.2929	3.4144		0.9554	10.7605
7	1	0.4393	0.4339	0.6175	1.4229	0.7028	0.6136		
	2	0.3040	0.7819	0.8389	0.7247	1.3798		0.7204	3.4077
	3	0.1085	0.9750	0.9810	0.2212	4.5208		0.9689	13.1578
	4	0.4876		0.4876			0.4876		
8	1	0.4268	0.1951	0.4693	1.8190	0.5498	0.3451		
	2	0.3168	0.5556	0.6396	0.9907	1.0094		0.4564	1.3041
	3	0.2418	0.8315	0.8659	0.5585	1.7906		0.7956	5.4126
	4	0.0849	0.9808	0.9845	0.1725	5.7978		0.9771	15.2977
9	1	0.3686	0.3420	0.5028	1.4661	0.6821	0.4844		
	2	0.3005	0.6428	0.7096	0.8470	1.1807		0.5682	2.3008
	3	0.1961	0.8661	0.8880	0.4417	2.2642		0.8436	7.3155
	4	0.0681	0.9848	0.9872	0.1380	7.2478		0.9824	17.2249
	5	0.3923		0.3923			0.3923		
10	1	0.3522	0.1564	0.3854	1.8279	0.5471	0.2814		
	2	0.3178	0.454	0.5542	1.1469	0.8719		0.3242	0.5412
	3	0.2522	0.7071	0.7507	0.6719	1.4884		0.6606	3.9742
	4	0.1619	0.891	0.9056	0.3576	2.7968		0.8762	9.0742
	5	0.0558	0.9877	0.9893	0.1128	8.8645		0.9861	18.9669

.01 dB Chebyshev Design Table Fig. 13

ORDER	SECTION	REAL PART	IMAGINARY PART	F _O	α	Q	-3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	0.7071	0.7071	1.0000	1.4142	0.7071	1.0000		
3	1	0.5000	0.8660	1.0000	1.0000	1.0000		0.7071	1.2493
	2	1.0000		1.0000			1.0000		
4	1	0.9239	0.3827	1.0000	1.8478	0.5412	0.7195		
	2	0.3827	0.9239	1.0000	0.7654	1.3065		0.8409	3.0102
5	1	0.8090	0.5878	1.0000	1.6180	0.6180	0.8588		
	2	0.3090	0.9511	1.0000	0.6180	1.6182		0.8995	4.6163
	3	1.0000		1.0000			1.0000		
6	1	0.9659	0.2588	1.0000	1.9319	0.5176	0.6758		
	2	0.7071	0.7071	1.0000	1.4142	0.7071	1.0000		
	3	0.2588	0.9659	1.0000	0.5176	1.9319		0.9306	6.0210
7	1	0.9010	0.4339	1.0000	1.8019	0.5550	0.7449		
	2	0.6235	0.7818	1.0000	1.2470	0.8019		0.4717	0.2204
	3	0.2225	0.9749	1.0000	0.4450	2.2471		0.9492	7.2530
	4	1.0000		1.0000			1.0000		
8	1	0.9808	0.1951	1.0000	1.9616	0.5098	0.6615		
	2	0.8315	0.5556	1.0000	1.6629	0.6013	0.8295		
	3	0.5556	0.8315	1.0000	1.1112	0.9000		0.6186	0.6876
	4	0.1951	0.9808	1.0000	0.3902	2.5628		0.9612	8.3429
9	1	0.9397	0.3420	1.0000	1.8794	0.5321	0.7026		
	2	0.7660	0.6428	1.0000	1.5320	0.6527	0.9172		
	3	0.5000	0.8660	1.0000	1.0000	1.0000		0.7071	1.2493
	4	0.1737	0.9848	1.0000	0.3474	2.8785		0.9694	9.3165
	5	1.0000		1.0000			1.0000		
10	1	0.9877	0.1564	1.0000	1.9754	0.5062	0.6549		
	2	0.8910	0.4540	1.0000	1.7820	0.5612	0.7564		
	3	0.7071	0.7071	1.0000	1.4142	0.7071	1.0000		
	4	0.4540	0.8910	1.0000	0.9080	1.1013		0.7667	1.8407
	5	0.1564	0.9877	1.0000	0.3128	3.1970		0.9752	10.2023

**.1 dB
Chebyshev
Design Table
Fig. 14**

ORDER	SECTION	REAL PART	IMAGINARY PART	F_o	α	Q	-3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	0.5621	0.7154	0.9098	1.2356	0.8093		0.4425	0.2502
3	1	0.3062	0.8712	0.9234	0.6632	1.5079		0.8156	4.0734
	2	0.6124		0.6124			0.6124		
4	1	0.4501	0.3840	0.5916	1.5215	0.6572	0.5470		
	2	0.1865	0.9272	0.9458	0.3944	2.5356		0.9082	8.2538
5	1	0.3247	0.5892	0.6727	0.9653	1.0359		0.4917	1.4585
	2	0.1240	0.9533	0.9613	0.2580	3.8763		0.9452	11.8413
	3	0.4013		0.4013			0.4013		
6	1	0.3284	0.2593	0.4184	1.5697	0.6371	0.3730		
	2	0.2404	0.7083	0.7480	0.6428	1.5557		0.6663	4.3121
	3	0.0880	0.9675	0.9715	0.1811	5.5205		0.9635	14.8753
7	1	0.2652	0.4344	0.5090	1.0421	0.9596		0.3441	1.0173
	2	0.1835	0.7828	0.8040	0.4565	2.1908		0.7610	7.0443
	3	0.0655	0.9761	0.9783	0.1339	7.4679		0.9739	17.4835
	4	0.2944		0.2944			0.2944		
8	1	0.2543	0.1953	0.3206	1.5862	0.6304	0.2822		
	2	0.2156	0.5561	0.5964	0.7230	1.3832		0.5126	3.4258
	3	0.1441	0.8323	0.8447	0.3412	2.9309		0.8197	9.4683
	4	0.0506	0.9817	0.9830	0.1029	9.7173		0.9804	19.7624
9	1	0.2176	0.3423	0.4056	1.0730	0.9320		0.2642	0.8624
	2	0.1774	0.6433	0.6673	0.5317	1.8808		0.6184	5.8052
	3	0.1158	0.8667	0.8744	0.2649	3.7755		0.8589	11.6163
	4	0.0402	0.9856	0.9864	0.0815	12.2659		0.9848	21.7812
	5	0.2315		0.2315			0.2315		
10	1	0.2065	0.1565	0.2591	1.5940	0.6274	0.2267		
	2	0.1863	0.4543	0.4910	0.7588	1.3178		0.4143	3.0721
	3	0.1478	0.7075	0.7228	0.4090	2.4451		0.6919	7.9515
	4	0.0949	0.8915	0.8965	0.2117	4.7236		0.8864	13.5344
	5	0.0327	0.9883	0.9888	0.0661	15.1199		0.9878	23.5957

**.25 dB
Chebyshev
Design Table
Fig. 15**

ORDER	SECTION	REAL PART	IMAGINARY PART	F_o	α	Q	-3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	0.6104	0.7106	0.9368	1.3032	0.7673		0.3638	0.0999
3	1	0.3490	0.8684	0.9359	0.7458	1.3408		0.7952	3.1978
	2	0.6970		0.6970			0.6970		
4	1	0.2177	0.9254	0.9507	0.4580	2.1834		0.8994	7.0167
	2	0.5257	0.3833	0.6506	1.6160	0.6188	0.5596		
5	1	0.3842	0.5884	0.7027	1.0935	0.9145		0.4457	0.7662
	2	0.1468	0.9521	0.9634	0.3048	3.2812		0.9407	10.4226
	3	0.4749		0.4749			0.4749		
6	1	0.3916	0.2590	0.4695	1.6682	0.5995	0.3879		
	2	0.2867	0.7077	0.7636	0.7509	1.3316		0.6470	3.1478
	3	0.1049	0.9667	0.9724	0.2158	4.6348		0.9610	13.3714
7	1	0.3178	0.4341	0.5380	1.1814	0.8464		0.2957	0.4157
	2	0.2200	0.7823	0.8126	0.5414	1.8469		0.7507	5.6595
	3	0.0785	0.9755	0.9787	0.1604	6.2335		0.9723	15.9226
	4	0.3528		0.3528			0.3528		
8	1	0.3058	0.1952	0.3628	1.6858	0.5932	0.2956		
	2	0.2529	0.5558	0.6106	0.8283	1.2073		0.4949	2.4532
	3	0.1732	0.8319	0.8497	0.4077	2.4531		0.8137	7.9784
	4	0.0608	0.9812	0.9831	0.1237	8.0819		0.9793	18.1669
9	1	0.2622	0.3421	0.4310	1.2166	0.8219		0.2197	0.3037
	2	0.2137	0.6430	0.6776	0.6308	1.5854		0.6064	4.4576
	3	0.1395	0.8663	0.8775	0.3180	3.1450		0.8550	10.0636
	4	0.0485	0.9852	0.9864	0.0982	10.1795		0.9840	20.1650
	5	0.2790		0.2790			0.2790		
10	1	0.2493	0.1564	0.2943	1.6942	0.5902	0.2382		
	2	0.2249	0.4541	0.5067	0.8876	1.1266		0.3945	1.9880
	3	0.1785	0.7073	0.7295	0.4894	2.0434		0.6844	6.4750
	4	0.1146	0.8913	0.8986	0.2551	3.9208		0.8839	11.9386
	5	0.0395	0.9880	0.9888	0.0799	12.5163		0.9872	21.9565

**.5 dB
Chebyshev
Design
Table
Fig. 16**

ORDER	SECTION	REAL PART	IMAGINARY PART	F ₀	α	Q	-3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	0.4508	0.7351	0.8623	1.0456	0.9564		0.5806	0.9995
3	1	0.2257	0.8822	0.9106	0.4957	2.0173		0.8528	6.3708
	2	0.4513		0.4513			0.4513		
4	1	0.3199	0.3868	0.5019	1.2746	0.7845		0.2174	0.1557
	2	0.1325	0.9339	0.9433	0.2809	3.5594		0.9245	11.1142
5	1	0.2265	0.5918	0.6337	0.7149	1.3988		0.5467	3.5089
	2	0.0865	0.9575	0.9614	0.1800	5.5559		0.9536	14.9305
	3	0.2800		0.2800			0.2800		
6	1	0.2268	0.2601	0.3451	1.3144	0.7608		0.1273	0.0813
	2	0.1550	0.7106	0.7273	0.4262	2.3462		0.6935	7.6090
	3	0.0608	0.9707	0.9726	0.1249	8.0036		0.9688	18.0827
7	1	0.1819	0.4354	0.4719	0.7710	1.2971		0.3956	2.9579
	2	0.1259	0.7846	0.7946	0.3169	3.1558		0.7744	10.0927
	3	0.0449	0.9785	0.9795	0.0918	10.8982		0.9775	20.7563
	4	0.2019		0.2019			0.2019		
8	1	0.1737	0.1956	0.2616	1.3280	0.7530		0.0899	0.0611
	2	0.1473	0.5571	0.5762	0.5112	1.9560		0.5373	6.1210
	3	0.0984	0.8337	0.8395	0.2344	4.2657		0.8279	12.6599
	4	0.0346	0.9836	0.9842	0.0702	14.2391		0.9830	23.0750
9	1	0.1482	0.3427	0.3734	0.7938	1.2597		0.3090	2.7498
	2	0.1208	0.6442	0.6554	0.3686	2.7129		0.6328	8.8187
	3	0.0788	0.8679	0.8715	0.1809	5.5268		0.8643	14.8852
	4	0.0274	0.9869	0.9873	0.0555	18.0226		0.9865	25.1197
	5	0.1577		0.1577			0.1577		
10	1	0.1403	0.1567	0.2103	1.3341	0.7496		0.0698	0.0530
	2	0.1266	0.4548	0.4721	0.5363	1.8645		0.4368	5.7354
	3	0.1005	0.7084	0.7155	0.2809	3.5597		0.7012	11.1147
	4	0.0645	0.8926	0.8949	0.1441	6.9374		0.8903	16.8466
	5	0.0222	0.9895	0.9897	0.0449	22.2916		0.9893	26.9650

**1 dB
Chebyshev
Design
Table
Fig. 17**

ORDER	SECTION	REAL PART	IMAGINARY PART	F ₀	α	Q	-3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	0.5129	0.7225	1.2314	1.1577	0.8638		0.7072	0.5002
3	1	0.2683	0.8753	1.0688	0.5861	1.7061		0.9727	5.0301
	2	0.5366		0.6265			0.6265		
4	1	0.3872	0.3850	0.5969	1.4182	0.7051	0.5951		
	2	0.1605	0.9297	1.0313	0.3402	2.9391		1.0010	9.4918
5	1	0.2767	0.5902	0.6905	0.8490	1.1779		0.5522	2.2849
	2	0.1057	0.9550	1.0178	0.2200	4.5451		1.0054	13.2037
	3	0.3420		0.3623			0.3623		
6	1	0.2784	0.2596	0.3963	1.4627	0.6836	0.3827		
	2	0.2037	0.7091	0.7680	0.5522	1.8109		0.7071	5.5025
	3	0.0746	0.9687	1.0114	0.1536	6.5119		1.0055	16.2998
7	1	0.2241	0.4349	0.5040	0.9161	1.0916		0.3839	1.7838
	2	0.1550	0.7836	0.8228	0.3881	2.5767		0.7912	8.3880
	3	0.0553	0.9771	1.0081	0.1130	8.8487		1.0049	18.9515
	4	0.2487		0.2562			0.2562		
8	1	0.2144	0.1955	0.2968	1.4779	0.6767	0.2835		
	2	0.1817	0.5565	0.5989	0.6208	1.6109		0.5381	4.5815
	3	0.1214	0.8328	0.8610	0.2885	3.4662		0.8429	10.8885
	4	0.0426	0.9824	1.0060	0.0867	11.5305		1.0041	21.2452
9	1	0.1831	0.3425	0.3954	0.9429	1.0605		0.2947	1.6023
	2	0.1493	0.6436	0.6727	0.4520	2.2126		0.6374	7.1258
	3	0.0974	0.8671	0.8884	0.2233	4.4779		0.8773	13.0759
	4	0.0338	0.9861	1.0046	0.0696	14.5829		1.0034	23.2820
	5	0.1949		0.1984			0.1984		
10	1	0.1736	0.1566	0.2338	1.4851	0.6734	0.2221		
	2	0.1566	0.4545	0.4807	0.6515	1.5349		0.4267	4.2087
	3	0.1243	0.7078	0.7186	0.3459	2.8907		0.6968	9.3520
	4	0.0798	0.8919	0.8955	0.1782	5.6107		0.8883	15.0149
	5	0.0275	0.9887	0.9891	0.0556	17.9833		0.9883	25.1008

Bessel Design Table Fig. 18

ORDER	SECTION	REAL PART	IMAGINARY PART	F _O	α	Q	-3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	1.0087	0.6680	1.2098	1.6675	0.5997	0.9999		
3	1	0.8541	1.0725	1.3710	1.2459	0.8026		0.6487	0.2232
	2	1.0459		1.0459			1.0459		
4	1	0.9648	0.4748	1.0753	1.7945	0.5573	0.8056		
	2	0.7448	1.4008	1.5865	0.9389	1.0650		1.1864	1.6286
5	1	0.8915	0.8733	1.2480	1.4287	0.6999	1.2351		
	2	0.6731	1.7085	1.8363	0.7331	1.3641		1.5703	3.3234
	3	0.9430		0.9430			0.9430		
6	1	0.8904	0.4111	0.9807	1.8158	0.5507	0.7229		
	2	0.8233	1.2179	1.4701	1.1201	0.8928		0.8975	0.6495
	3	0.6152	1.9810	2.0743	0.5932	1.6859		1.8831	4.9365
7	1	0.8425	0.7791	1.1475	1.4684	0.6810	1.1036		
	2	0.7708	1.5351	1.7177	0.8975	1.1143		1.3276	1.9162
	3	0.5727	2.2456	2.3175	0.4942	2.0233		2.1713	6.3948
	4	0.8615		0.8615			0.8615		
8	1	0.8195	0.3711	0.8996	1.8219	0.5489	0.6600		
	2	0.7930	1.1054	1.3604	1.1658	0.8578		0.7701	0.4705
	3	0.7213	1.8134	1.9516	0.7392	1.3528		1.6638	3.2627
	4	0.5341	2.4761	2.5330	0.4217	2.3713		2.4178	7.6973
9	1	0.7853	0.7125	1.0604	1.4812	0.6751	1.0102		
	2	0.7555	1.4127	1.6020	0.9432	1.0602		1.1937	1.6005
	3	0.6849	2.0854	2.1950	0.6241	1.6024		1.9697	4.5404
	4	0.5060	2.7133	2.7601	0.3667	2.7274		2.6657	8.8633
	5	0.7983		0.7983			0.7983		
10	1	0.7592	0.3413	0.8324	1.8241	0.5482	0.6096		
	2	0.7467	1.0195	1.2637	1.1818	0.8462		0.6941	0.4145
	3	0.7159	1.6836	1.8295	0.7826	1.2778		1.5238	2.8507
	4	0.6475	2.3198	2.4085	0.5377	1.8598		2.2276	5.7152
	5	0.4777	2.9128	2.9517	0.3237	3.0895		2.8734	9.9130

Equiripple with .05° Error Design Table Fig. 19

ORDER	SECTION	REAL PART	IMAGINARY PART	F _O	α	Q	-3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	1.1050	0.6368	1.2754	1.7328	0.5771	1.0020		
3	1	1.0509	1.0025	1.4524	1.4471	0.6910	1.4185		
	2	1.3270		1.3270			1.3270		
4	1	1.3596	0.4071	1.4192	1.9160	0.5219	0.9705		
	2	0.9877	1.2476	1.5912	1.2414	0.8055		0.7622	0.2349
5	1	1.3851	0.7201	1.5611	1.7745	0.5635	1.1876		
	2	0.9606	1.4756	1.7607	1.0911	0.9165		1.1201	0.7768
	3	1.5069		1.5069			1.5069		
6	1	1.5735	0.3213	1.6060	1.9596	0.5103	1.0638		
	2	1.3836	0.9727	1.6913	1.6361	0.6112	1.4323		
	3	0.9318	1.6640	1.9071	0.9772	1.0234		1.3786	1.3851
7	1	1.6130	0.5896	1.7174	1.8784	0.5324	1.2074		
	2	1.3797	1.1923	1.8235	1.5132	0.6608	1.6964		
	3	0.9104	1.8375	2.0507	0.8879	1.1262		1.5961	1.9860
	4	1.6853		1.6853			1.6853		
8	1	1.7627	0.2737	1.7838	1.9763	0.5060	1.1675		
	2	0.8955	2.0044	2.1953	0.8158	1.2258		1.7932	2.5585
	3	1.3780	1.3926	1.9591	1.4067	0.7109		0.2011	0.0005
	4	1.6419	0.8256	1.8378	1.7868	0.5597	1.3849		
9	1	1.8081	0.5126	1.8794	1.9242	0.5197	1.2774		
	2	1.6532	1.0319	1.9488	1.6966	0.5894	1.5747		
	3	1.3683	1.5685	2.0915	1.3148	0.7606		0.7668	0.0807
	4	0.8788	2.1509	2.3235	0.7564	1.3220		1.9632	3.0949
	5	1.8575		1.8575			1.8575		
10	1	1.9335	0.2451	1.9490	1.9841	0.5040	1.2685		
	2	1.8467	0.7335	1.9870	1.8587	0.5380	1.4177		
	3	1.6661	1.2246	2.0678	1.6115	0.6205	1.7848		
	4	1.3648	1.7395	2.2110	1.2346	0.8100		1.0785	0.2531
	5	0.8686	2.2994	2.4580	0.7067	1.4150		2.1291	3.5944

**Equiripple
with .5° Error
Design Table
Fig. 20**

ORDER	SECTION	REAL PART	IMAGINARY PART	F _O	α	Q	-3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
3	1	0.9622	1.2214	1.5549	1.2377	0.8080		0.7523	0.2448
	2	0.9776	0.5029	1.0994	1.7785	0.5623	0.8338		
4	1	0.7940	0.5029	0.9399	1.6896	0.5919	0.7636		
	2	0.6304	1.5407	1.6647	0.7574	1.3203		1.4058	3.0859
5	1	0.6190	0.8254	1.0317	1.1999	0.8334		0.5460	0.3548
	2	0.3559	1.5688	1.6087	0.4425	2.2600		1.5279	7.3001
	3	0.6650		0.6650			0.6650		
6	1	0.5433	0.3431	0.6426	1.6910	0.5914	0.5215		
	2	0.4672	0.9991	1.1029	0.8472	1.1804		0.8831	2.2992
	3	0.2204	1.5067	1.5227	0.2895	3.4545		1.4905	10.8596
7	1	0.4580	0.5932	0.7494	1.2223	0.8182		0.3770	0.2874
	2	0.3649	1.1286	1.1861	0.6153	1.6253		1.0680	4.6503
	3	0.1522	1.4938	1.5015	0.2027	4.9328		1.4860	13.9067
	4	0.4828		0.4828			0.4828		
8	1	0.4222	0.2640	0.4979	1.6958	0.5897	0.4026		
	2	0.3833	0.7716	0.8616	0.8898	1.1239		0.6697	1.9722
	3	0.2678	1.2066	1.2360	0.4333	2.3076		1.1765	7.4721
	4	0.1122	1.4798	1.4840	0.1512	6.6134		1.4755	16.4334
9	1	0.3700	0.4704	0.5985	1.2365	0.8088		0.2905	0.2480
	2	0.3230	0.9068	0.9626	0.6711	1.4901		0.8473	3.9831
	3	0.2309	1.2634	1.2843	0.3596	2.7811		1.2421	9.0271
	4	0.0860	1.4740	1.4765	0.1165	8.5804		1.4715	18.6849
	5	0.3842		0.3842			0.3842		
10	1	0.3384	0.2101	0.3983	1.6991	0.5885	0.3212		
	2	0.3164	0.6180	0.6943	0.9114	1.0972		0.5309	1.8164
	3	0.2677	0.9852	1.0209	0.5244	1.9068		0.9481	5.9157
	4	0.1849	1.2745	1.2878	0.2871	3.4825		1.2610	10.9284
	5	0.0671	1.4389	1.4405	0.0931	10.7401		1.4373	20.6296

**Gaussian to
12 dB
Design Table
Fig. 21**

ORDER	SECTION	REAL PART	IMAGINARY PART	F _O	α	Q	-3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
2	1	0.8590	0.6981	1.1063	1.5521	0.6443	1.0000		
3	1	0.6969	1.1318	1.3292	1.0486	0.9536		0.8918	0.9836
	2	0.8257		0.8257			0.8257		
4	1	0.7448	0.5133	0.9045	1.6468	0.6072	0.7597		
	2	0.6037	1.4983	1.6154	0.7475	1.3379		1.3713	3.1817
5	1	0.6775	0.9401	1.1588	1.1693	0.8552		0.6518	0.4579
	2	0.5412	1.8256	1.9041	0.5684	1.7592		1.7435	5.2720
	3	0.7056		0.7056			0.7056		
6	1	0.6519	0.4374	0.7850	1.6608	0.6021	0.6522		
	2	0.6167	1.2963	1.4355	0.8592	1.1639		1.1402	2.2042
	3	0.4893	2.0982	2.1545	0.4542	2.2016		2.0404	7.0848
7	1	0.6190	0.8338	1.0385	1.1922	0.8388		0.5586	0.3798
	2	0.5816	1.6455	1.7453	0.6665	1.5004		1.5393	4.0353
	3	0.4598	2.3994	2.4431	0.3764	2.6567		2.3549	8.6433
	4	0.6283		0.6283			0.6283		
8	1	0.5791	0.3857	0.6958	1.6646	0.6007	0.5764		
	2	0.5665	1.1505	1.2824	0.8835	1.1319		1.0014	2.0187
	3	0.5303	1.8914	1.9643	0.5399	1.8521		1.8155	5.6819
	4	0.4148	2.5780	2.6112	0.3177	3.1475		2.5444	10.0703
9	1	0.5688	0.7595	0.9489	1.1989	0.8341		0.5033	0.3581
	2	0.5545	1.5089	1.6076	0.6899	1.4496		1.4033	3.7748
	3	0.5179	2.2329	2.2922	0.4519	2.2130		2.1720	7.1270
	4	0.4080	2.9028	2.9313	0.2784	3.5923		2.8740	11.1925
	5	0.5728		0.5728			0.5728		
10	1	0.5249	0.3487	0.6302	1.6659	0.6003	0.5215		
	2	0.5193	1.0429	1.1650	0.8915	1.1217		0.9044	1.9598
	3	0.5051	1.7264	1.7988	0.5616	1.7806		1.6509	5.3681
	4	0.4711	2.3850	2.4311	0.3876	2.5802		2.3380	8.3994
	5	0.3708	2.9940	3.0169	0.2458	4.0681		2.9709	12.2539

**Gaussian
to 6 dB
Design Table
Fig. 22**

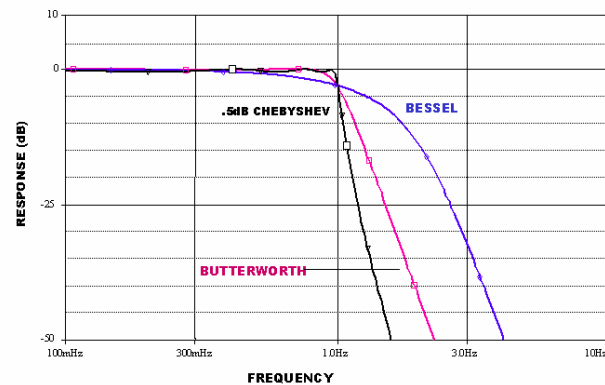
ORDER	SECTION	REAL PART	IMAGINARY PART	F _o	α	Q	-3 dB FREQUENCY	PEAKING FREQUENCY	PEAKING LEVEL
3	1	0.9360	1.2168	1.5352	1.2194	0.8201		0.7775	0.2956
	2	0.9360		0.9360			0.9360		
4	1	0.9278	1.6995	1.9363	0.9583	1.0435		1.4239	1.5025
	2	0.9192	0.5560	1.0743	1.7113	0.5844	0.8582		
5	1	0.9075	0.9973	1.2832	1.2585	0.7946		0.5853	0.1921
	2	0.7153	0.2053	0.7442	1.9224	0.5202	0.5065		
	3	0.8131		0.8131			0.8131		
6	1	0.7019	0.4322	0.8243	1.7030	0.5872	0.6627		
	2	0.6667	1.2931	1.4549	0.9165	1.0911		1.1080	1.7809
	3	0.4479	2.1363	2.1827	0.4104	2.4366		2.0888	7.9227
7	1	0.6155	0.7703	0.9860	1.2485	0.8010		0.4632	0.2168
	2	0.5486	1.5154	1.6116	0.6808	1.4689		1.4126	3.8745
	3	0.2905	2.1486	2.1681	0.2680	3.7318		2.1289	11.5169
	4	0.6291		0.6291			0.6291		
8	1	0.5441	0.3358	0.6394	1.7020	0.5876	0.5145		
	2	0.5175	0.9962	1.1226	0.9220	1.0846		0.8512	1.7432
	3	0.4328	1.6100	1.6672	0.5192	1.9260		1.5507	5.9962
	4	0.1978	2.0703	2.0797	0.1902	5.2571		2.0608	14.4545
9	1	0.4961	0.6192	0.7934	1.2505	0.7997		0.3705	0.2116
	2	0.4568	1.2145	1.2976	0.7041	1.4203		1.1253	3.6221
	3	0.3592	1.7429	1.7795	0.4037	2.4771		1.7055	8.0594
	4	0.1489	2.1003	2.1056	0.1414	7.0704		2.0950	17.0107
	5	0.5065		0.5065			0.5065		
10	1	0.4535	0.2794	0.5327	1.7028	0.5873	0.4283		
	2	0.4352	0.8289	0.9362	0.9297	1.0756		0.7055	1.6904
	3	0.3886	1.3448	1.3998	0.5552	1.8011		1.2874	5.4591
	4	0.2908	1.7837	1.8072	0.3218	3.1074		1.7598	9.9618
	5	0.1136	2.0599	2.0630	0.1101	9.0802		2.0568	19.1751

COMPARING THE RESPONSES

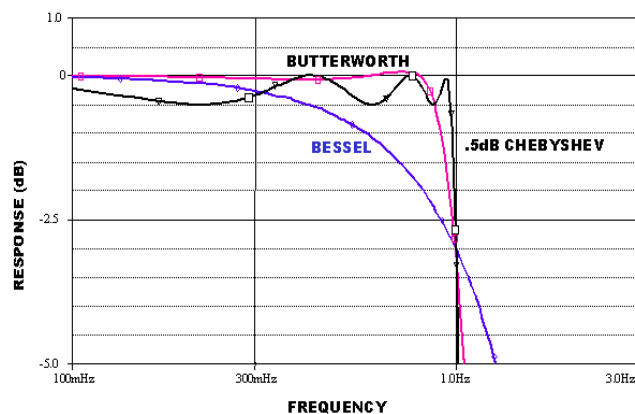
The responses of several all pole filters, namely the Bessel, Butterworth and Chebyshev (in this case of .5 dB ripple) will now be compared. An 8 pole filter is used as the basis for the comparison.

The responses have been normalized for a cutoff of 1 Hz. Comparing figs. 23 and 24, it is easy to see the tradeoffs in the various responses. Moving from Bessel through Butterworth to Chebyshev, notice that the amplitude discrimination improves as the transient behavior gets progressively poorer.

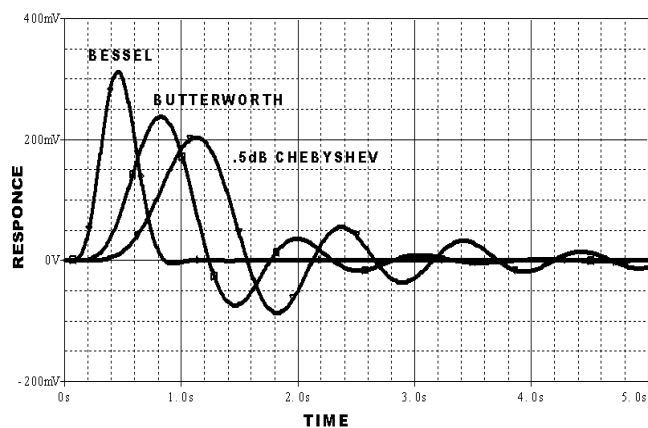
Amplitude Response Comparison Fig. 23



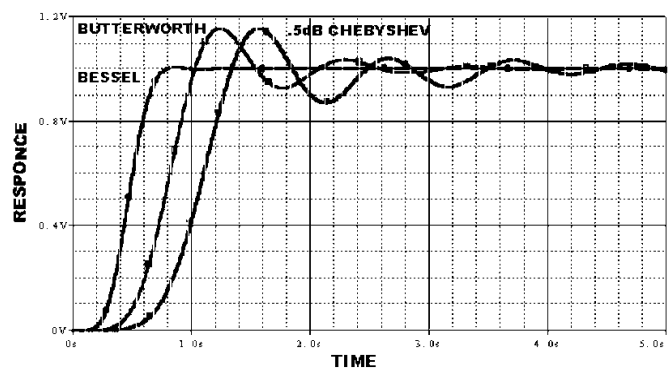
Amplitude Response Comparison (Detail) Fig. 24



Impulse Response Comparison Fig. 25



Step Response Comparison Fig. 26



FILTER TOPOLOGIES

Now that it has been decided what to build, it now must be decided how to build it. That means that it is necessary to decide which of the filter topologies to use. Filter design is a two step process where it is determined what is to be built (the filter transfer function) and then how to build it (the topology used for the circuit). In general, filters are built out of one pole sections, for real poles, and two pole sections, for pole pairs. While you can build a filter out of three pole, or higher order, sections, the interaction between the components in the sections increases and, therefore, component sensitivities go up. It is better to use buffers to isolate the various sections. In addition, it is assumed that all filter sections are driven from a low impedance source. Any source impedance can be modeled as being in series with the filter input.

In all of the design equation figures the following convention will be used:

H = circuit gain in the passband or at resonance

F_C = cutoff or resonant frequency in Hertz

ω_0 = cutoff or resonant frequency in radians/sec.

Q = circuit “quality factor”. Indicates circuit peaking.

$\alpha = 1/Q$ = damping ratio

It is unfortunate that the symbol α is used for damping ratio. It is not the same as the α that is used to denote pole locations ($\alpha \pm j\beta$). The same issue occurs for Q . It is used for the circuit quality factor and also the component

quality factor, which are not the same thing. The circuit Q is the amount of peaking in the circuit. This is a function of the angle of the pole to the origin in the s plane. The component Q is the amount of losses in what should be lossless reactances. These losses are the parasitics of the components; dissipation factor, leakage resistance, ESR (equivalent series resistance), etc. in capacitors and series resistance and parasitic capacitances in inductors.

SALLEN - KEY

The Sallen - Key configuration, also known as a voltage control voltage source (VCVS), was first introduced in 1955 by R. P. Sallen and E. L. Key of MIT's Lincoln Labs. It is one of the most widely used filter topologies. One reason for this popularity is that this configuration shows the least dependence of filter performance on the performance of the op amp. This is due to the fact that the op amp is configured as an amplifier, as opposed to an integrator, which minimizes the gain-bandwidth requirements of the op amp. This infers that for a given op amp, you will be able to design a higher frequency filter than with other topologies since the op amp gain bandwidth product will not limit the performance of the filter as it would if it were configured as an integrator. The signal phase through the filter is maintained (noninverting configuration). Another advantage of this configuration is that the ratio of the largest resistor value to the smallest resistor value and the ratio of the largest

capacitor value to the smallest capacitor value (component spread) are low, which is good for manufacturability. The frequency and Q terms are somewhat independent, but they are very sensitive to the gain parameter. The Sallen-Key is very Q-sensitive to element values, especially for high Q sections. The design equations for the Sallen-Key lowpass are shown in Fig. 23.

There is a special case of the 2nd order Sallen-Key lowpass filter. If the gain is set to 2, the capacitor values, as well as the resistor values, will be the same.

While the Sallen-Key filter is widely used, a serious drawback is that the filter is not easily tuned, due to interaction of the component values on F_0 and Q.

The design equations for the Sallen-Key highpass are shown in Fig. 24.

The bandpass case of the Sallen-Key filter has a limitation. The value of Q will determine the gain of the filter, i.e. it can not be set independent, as in the lowpass or highpass cases. The design equations for the Sallen-Key bandpass are shown in Fig. 25.

MULTIPLE FEEDBACK

The multiple feedback filter uses an op amp as an integrator. Therefore, the dependence of the transfer function on the op amp parameters is greater than in the Sallen-Key realization. It is hard to generate high Q, high frequency sections due to the limitations of the open loop gain of the op amp. A rule of thumb is that the open loop gain of the op amp should be at least 20 dB (x10) above the amplitude response at the resonant (or cutoff) frequency, including the peaking

caused by the Q of the filter. The peaking due to Q will cause an amplitude, A_0 :

$$A_0 = H Q \quad (2)$$

where H is the gain of the circuit. The multiple feedback filter will invert the phase of the signal. This is equivalent to adding the resulting 180° phase shift to the phase shift of the filter itself.

The maximum to minimum component value ratios is higher in the multiple feedback case than in the Sallen-Key realization. The design equations for the multiple feedback lowpass are given in Fig. 26.

The design equations for the multiple feedback highpass are given in Fig. 27.

The design equations for the multiple feedback bandpass case are given in Fig. 28. This circuit is widely used in low Q (<20) applications. It allows some tuning of the resonant frequency, F_0 , by making R2 variable. Q can be adjusted (with R5) as well, but this will also change F_0 . Tuning of F_0 can be accomplished by monitoring the output of the filter with the horizontal channel of an oscilloscope, with the input to the filter connected to the vertical channel. The display will be a Lissajous pattern. This pattern will be an ellipse that will collapse to a straight line at resonance, since the phase shift will be 180°. You could also adjust the output for maximum output, which will also occur at resonance, but this is usually not as precise, especially at lower values of Q where there is a less pronounced peak.

STATE VARIABLE

The state-variable realization offers the most precise implementation, at the expense of many more circuit elements. All three major parameters (gain, Q & ω_0) may be adjusted independently, and lowpass, highpass, and bandpass outputs are available simultaneously. Note that the lowpass and highpass outputs are inverted in phase while the bandpass output maintains the phase. The gain of each of the outputs of the filter is also independently variable.

Since all parameters of the state variable filter can be adjusted independently, component spread can be minimized. Also, variations due to temperature and component tolerances are minimized. The op amps used in the integrator sections will have the same limitations on op amp gain-bandwidth as described in the multiple feedback section.

The design equations for the state variable filter are shown in Fig. 29.

Tuning the resonant frequency of a state variable filter is accomplished by varying R_4 and R_5 . While you don't have to tune both, if you are varying over a wide range it is generally preferable. Holding R_1 constant, tuning R_2 sets the lowpass gain and tuning R_3 sets the highpass gain. Bandpass gain and Q are set by the ratio of R_6 & R_7 .

Since the parameters of a state variable filter are independent and tunable, it is easy to add electronic control of frequency, Q and ω_0 . This adjustment is accomplished by using multiplying DACs (MDACs) or digital pots. For the

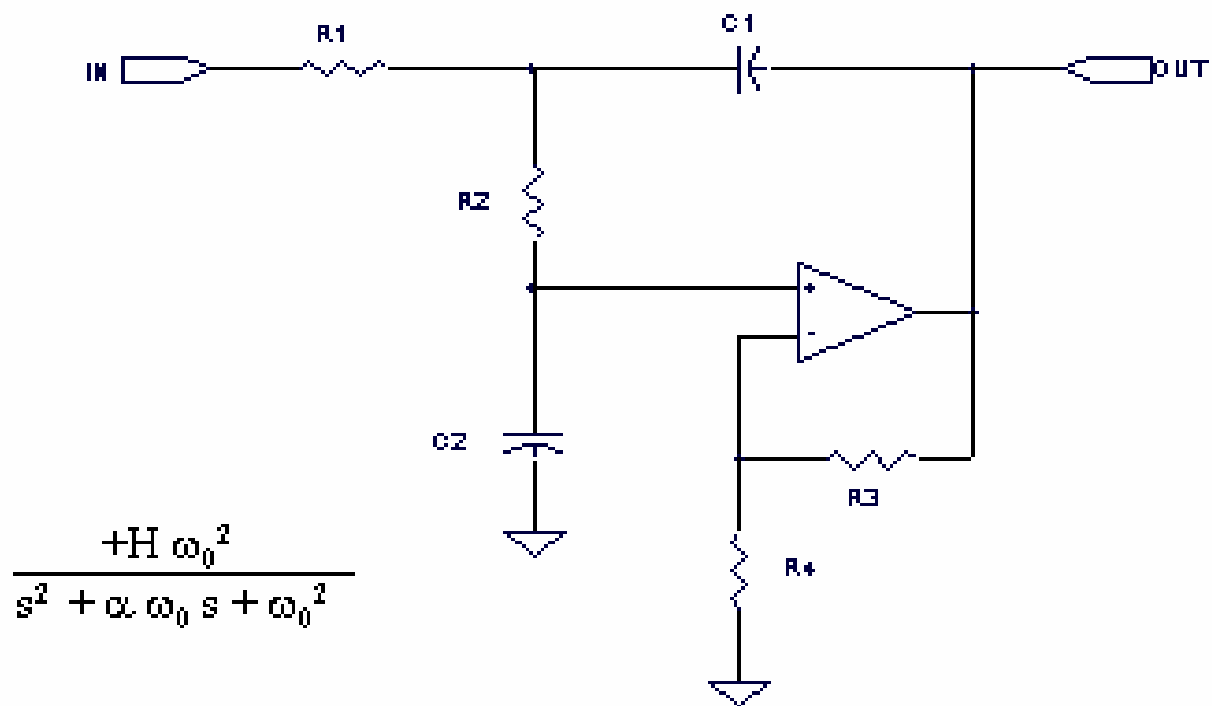
integrator sections adding the MDAC effectively increases the time constant by dividing the voltage driving the resistor, which, in turn, provides the charging current for the integrator capacitor. This in effect raises the resistance and, in turn, the time constant. The Q and gain can be varied by changing the ratio of the various feedback paths. A digital pot will accomplish the same feat in a more direct manner, by directly changing the resistance value. The resultant tunable filter offers a great deal of utility in measurement and control circuitry.

BIQUADRATIC (BIQUAD)

A close cousin of the state variable filter is the biquad. The name of this circuit was first used by J. Tow in 1968 and later by L. C. Thomas in 1971. The name derives from the fact that the transfer function is a quadratic function in both the numerator and the denominator. Hence the transfer function is a biquadratic function. This circuit is a slight rearrangement of the state variable circuit. One significant difference is that there is not a separate highpass output. The bandpass output inverts the phase. There are two lowpass outputs, one in phase and one out of phase. With the addition of a fourth amplifier section, a highpass filter may be realized. The design equations for the biquad are given in Fig. 30.

Referring to Fig. 30, for the highpass output, the input, bandpass and second lowpass outputs are summed. In this case the constraints are that $R_1 = R_2 = R_3$ and $R_7 = R_8 = R_9$.

SALLEN-KEY LOWPASS



$$\frac{+H \omega_0^2}{s^2 + \alpha \omega_0 s + \omega_0^2}$$

$$\frac{V_O}{V_{IN}} = \frac{H \frac{1}{R_1 R_2 C_1 C_2}}{s^2 + s \left[\frac{C_2}{R_1} + \frac{C_2}{R_2} + (1-H) \frac{C_1}{R_2} \right] + \frac{1}{R_1 R_2 C_1 C_2}}$$

CHOOSE: C1

R3

THEN: $k = 2 \pi F_O C_1$

$$R_4 = \frac{R_3}{(H-1)}$$

$$m = \frac{\alpha^2}{4} + (H-1)$$

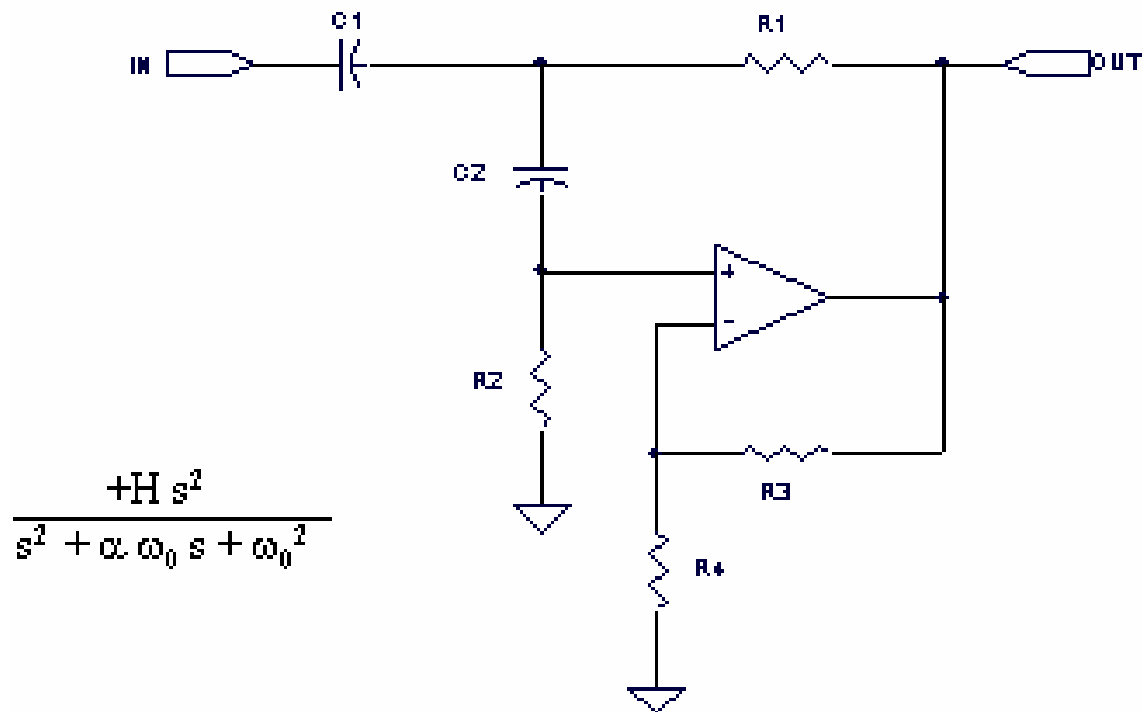
$$C_2 = m C_1$$

$$R_1 = \frac{2}{\alpha k}$$

$$R_2 = \frac{\alpha}{2mk}$$

Sallen-Key Lowpass Design Equations
Figure 27

SALLEN-KEY HIGHPASS



$$\frac{+H s^2}{s^2 + \alpha \omega_0 s + \omega_0^2}$$

$$\frac{V_O}{V_{IN}} = \frac{H s^2}{s^2 + s \left[\frac{\frac{C2}{R2} + \frac{C1}{R2} + (1-H) \frac{C2}{R1}}{C1 C2} \right] + \frac{1}{R1 R2 C1 C2}}$$

CHOOSE: $C1$ $R3$

THEN: $k = 2 \pi F_O C1$ $R4 = \frac{R3}{(H-1)}$

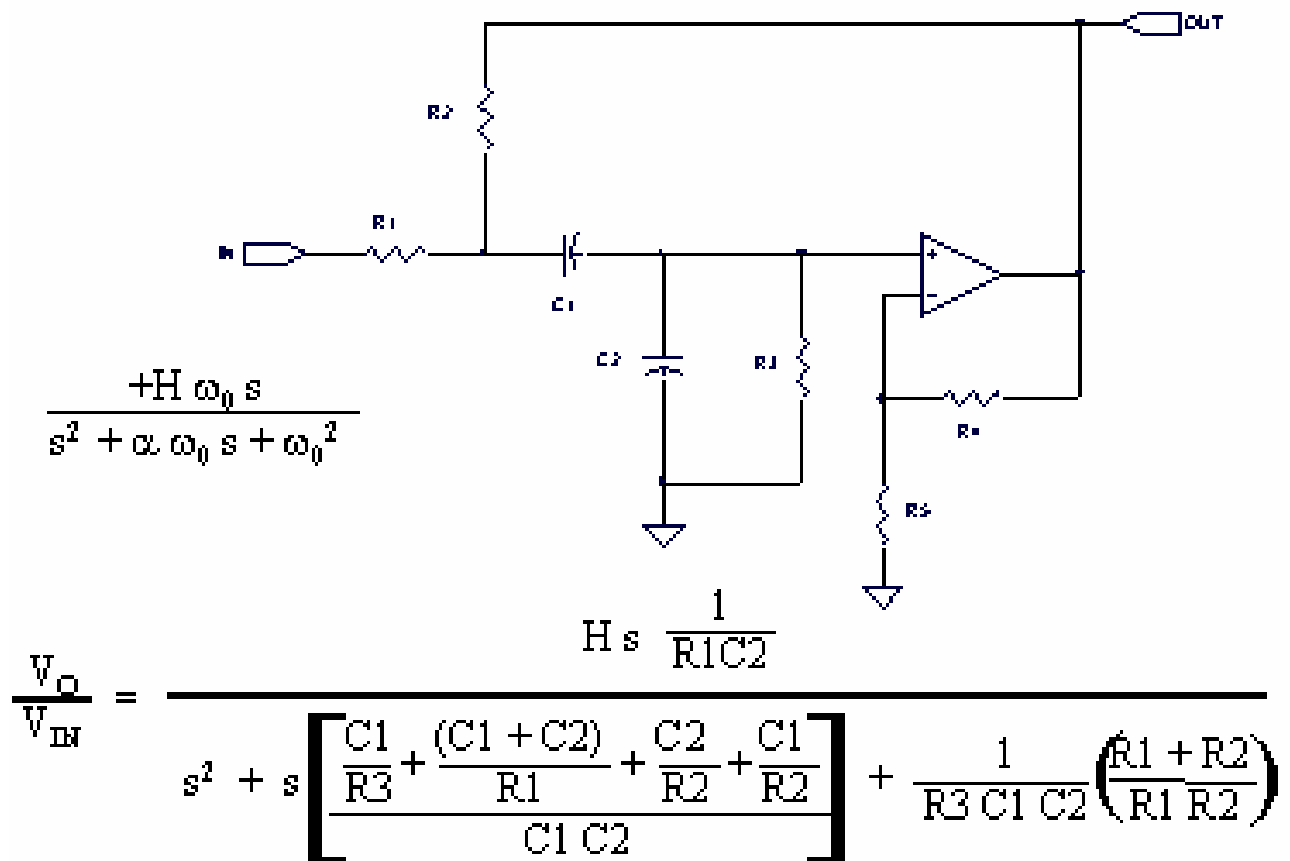
$$C2 = C1$$

$$R1 = \frac{\alpha + \sqrt{\alpha^2 + (H-1)}}{4k}$$

$$R2 = \frac{4}{\alpha + \sqrt{\alpha^2 + (H-1)}} * \frac{1}{k}$$

Sallen-Key Highpass Design Equations
Figure 28

SALLEN-KEY BANDPASS



CHOOSE: C_1 R_4

THEN: $k = 2 \pi F_O C_1$

$$R_5 = \frac{R_4}{(H-1)}$$

$$C_2 = \frac{1}{2} C_1$$

$$R_1 = \frac{2}{k}$$

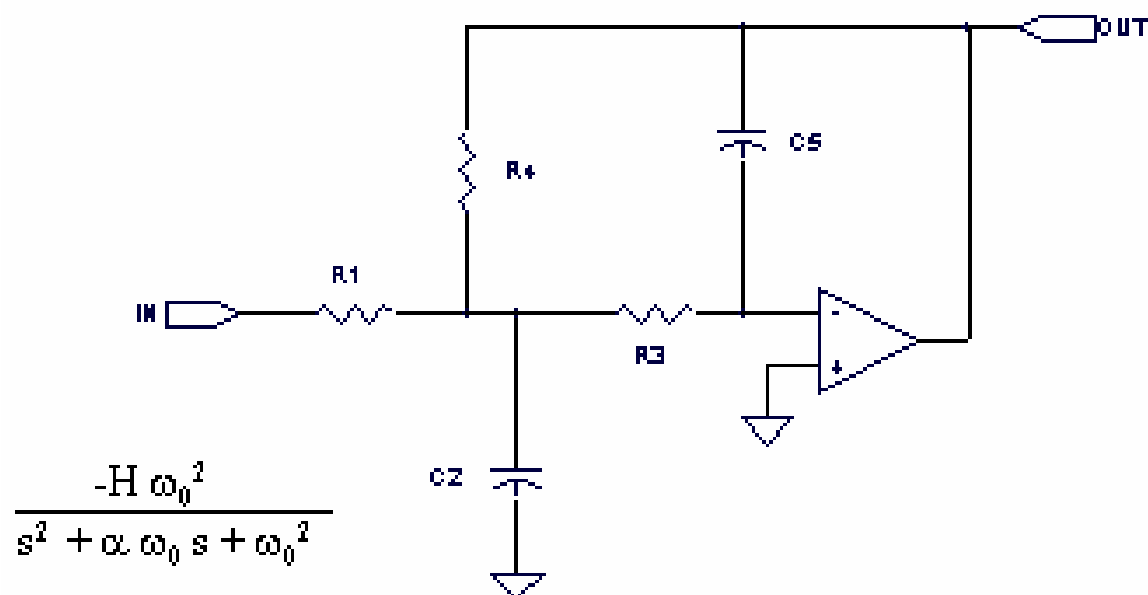
$$R_2 = \frac{2}{3k}$$

$$R_3 = \frac{4}{k}$$

$$H = \frac{1}{3} \left(6.5 - \frac{1}{Q} \right)$$

Sallen-Key Bandpass Design Equations
Figure 29

MULTIPLE FEEDBACK LOWPASS



$$\frac{V_O}{V_{IN}} = \frac{-H \frac{1}{R_3 R_4 C_2 C_5}}{s^2 + s \frac{1}{C_2} \left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} \right) + \frac{1}{R_3 R_4 C_2 C_5}}$$

CHOOSE: C5

THEN: $k = 2 \pi F_O C_5$

$$C_2 = \frac{4}{\alpha^2} (H + 1) C_5$$

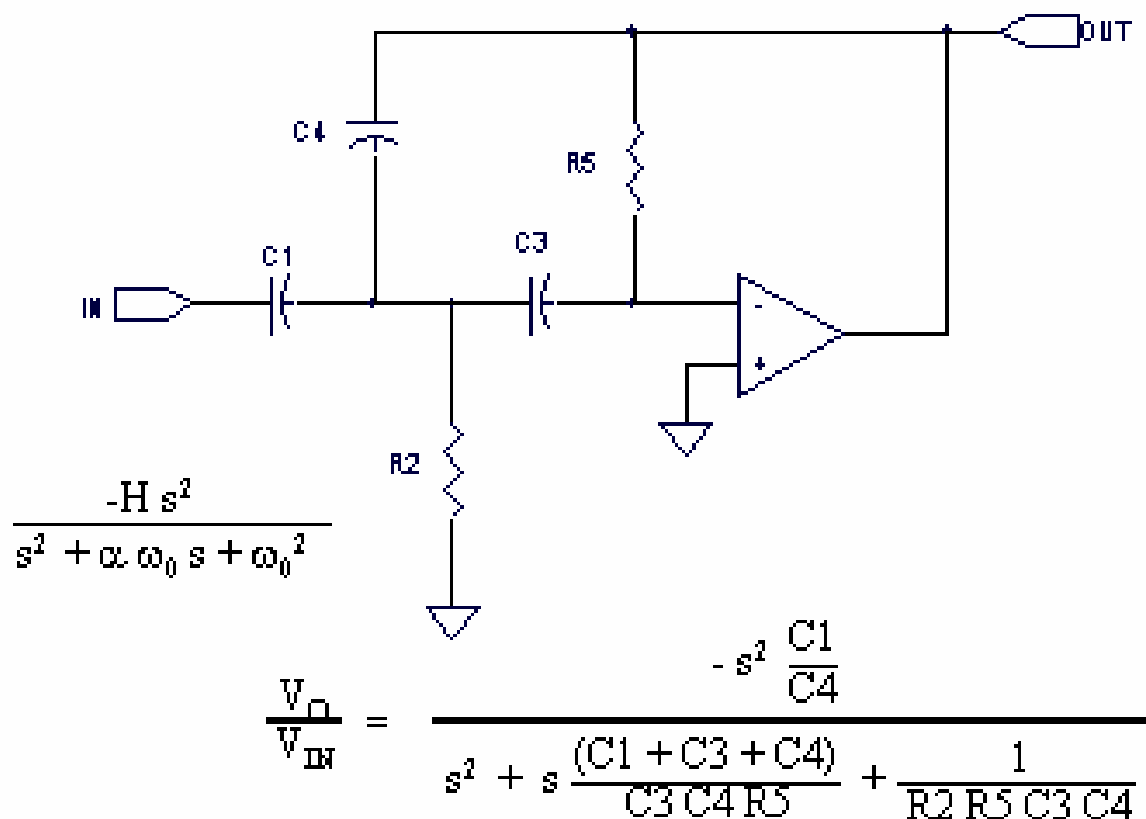
$$R_1 = \frac{\alpha}{2 H k}$$

$$R_3 = \frac{\alpha}{2 (H + 1) k}$$

$$R_4 = \frac{\alpha}{2 k}$$

Multiple Feedback Lowpass Design Equations
Figure 30

MULTIPLE FEEDBACK HIGHPASS



CHOOSE: C1

THEN: $k = 2 \pi F_O C1$

$$C3 = C1$$

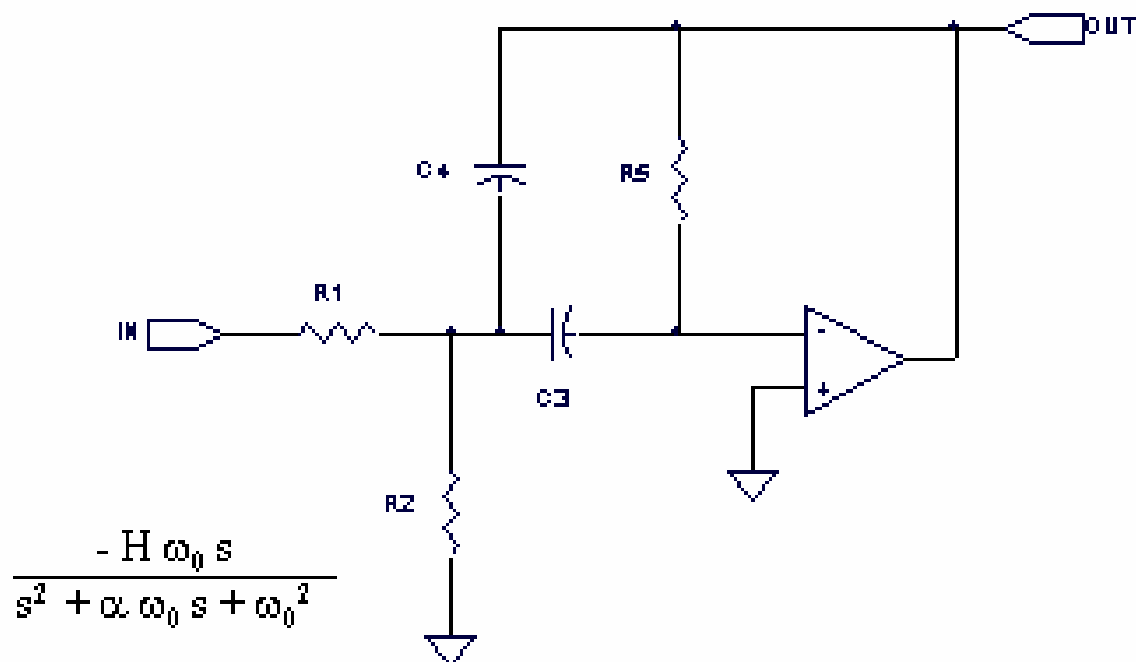
$$C4 = \frac{C1}{H}$$

$$R2 = \frac{\alpha}{k \left(2 + \frac{1}{H} \right)}$$

$$R5 = \frac{H \left(2 + \frac{1}{H} \right)}{\alpha k}$$

Multiple Feedback Highpass Design Equations
Figure 31

MULTIPLE FEEDBACK BANDPASS



$$\frac{-H \omega_0 s}{s^2 + \alpha \omega_0 s + \omega_0^2}$$

$$\frac{V_O}{V_{IN}} = \frac{-s \frac{1}{R1 C4}}{s^2 + s \frac{(C3 + C4)}{C3 C4 R5} + \frac{1}{R5 C3 C4} \left(\frac{1}{R1} + \frac{1}{R2} \right)}$$

CHOOSE: $C3$

THEN: $k = 2 \pi F_O C3$

$$C4 = C3$$

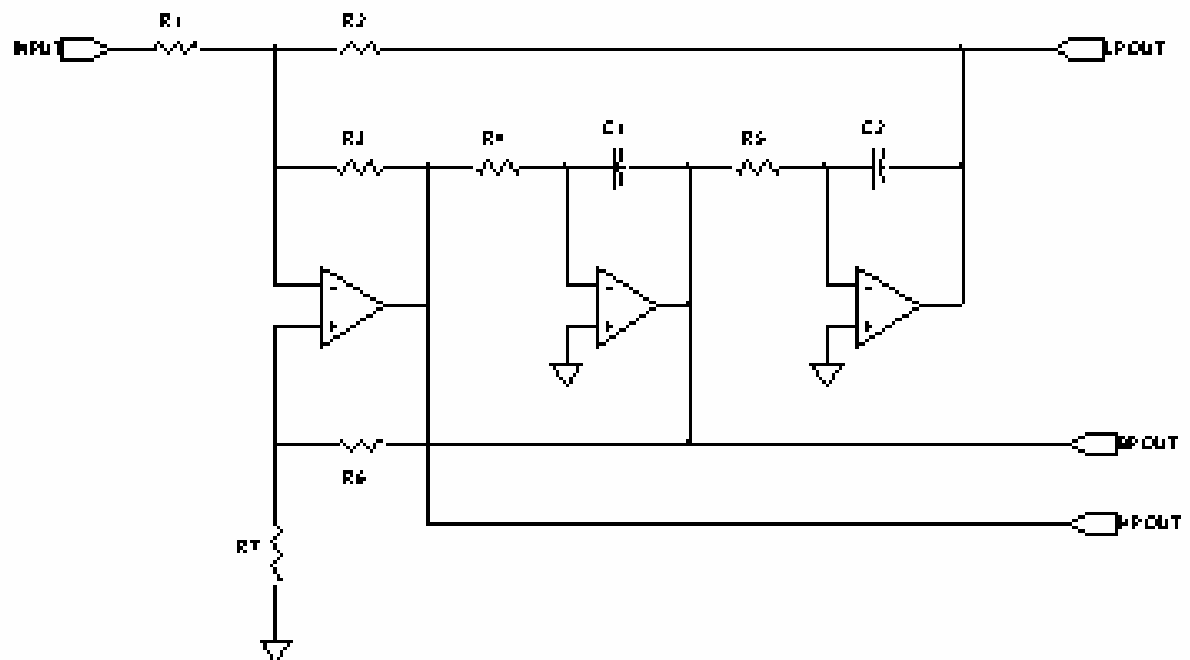
$$R1 = \frac{1}{H k}$$

$$R2 = \frac{1}{(2Q - H) k}$$

$$R5 = \frac{2Q}{k}$$

Multiple Feedback Bandpass Design Equations
Figure 32

STATE VARIABLE



$$A_{LP}(f=0) = -\frac{R_2}{R_1}$$

$$A_{HP}(f=\infty) = -\frac{R_3}{R_1}$$

$$\omega_0 = \sqrt{\frac{R_3}{R_2 R_4 R_5 C_1 C_2}}$$

CHOOSE R_1 :

$$R_2 = A_{LP} R_1$$

$$R_3 = A_{HP} R_1$$

CHOOSE C :

$$R = \frac{2\pi F_0}{C} \sqrt{\frac{A_{HP}}{A_{LP}}}$$

LET $R_4 = R_5 = R$, $C_1 = C_2 = C$

$$A_{BP}(f=\omega_0) = \frac{\frac{R_6 + R_7}{R_7}}{R_1 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)}$$

CHOOSE R_7 :

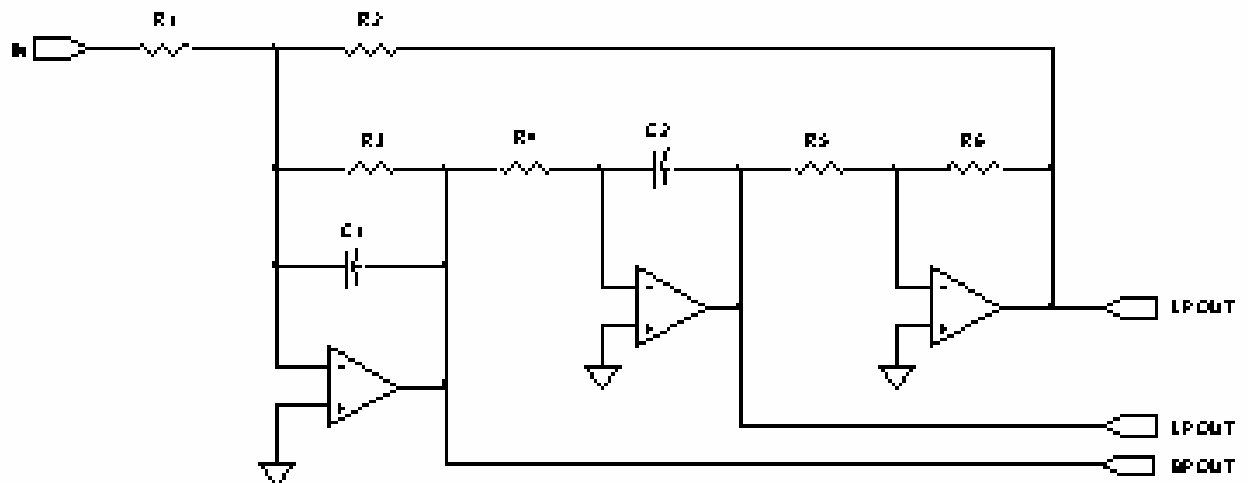
$$R_6 =$$

$$R_7 \sqrt{R_2 R_3} Q \left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \right)$$

State Variable Design Equations

Figure 33

BIQUADRATIC



CHOOSE C, R2, R5

$$K = 2 \pi f_0 C$$

$$C1 = C2 = C$$

$$R1 = \frac{R2}{H}$$

$$R3 = \frac{1}{k \alpha}$$

$$R4 = \frac{1}{k^2 R2}$$

CHOOSE C, R5, R7

$$K = 2 \pi f_0 C$$

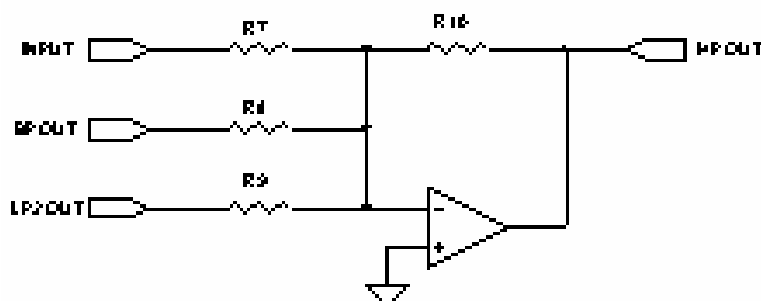
$$C1 = C2 = C$$

$$R1 = R2 = R3 = \frac{1}{k \alpha}$$

$$R4 = \frac{1}{k^2 R2}$$

$$R6 = R5$$

HIGHPASS



$$R7 = R8 = R9 = R$$

$$R10 = \frac{R}{H}$$

Biquad Design Equations
Figure 34

Like the state variable, the biquad filter is tunable. Adjusting R3 will adjust the Q. Adjusting R4 will set the resonant frequency. Adjusting R1 will set the gain. Frequency would generally be

adjusted first followed by Q and then gain. Setting the parameters in this manner minimizes the effects of component value interaction.

Op Amp Requirements

The curves that were generated for the prototype responses were done using an “ideal” op amp. In reality, the op amp is a single pole low pass filter. The amplifier’s dominate pole is the “corner Frequency” of the filter. The op amp’s transfer function would be added to the filter response.

What this means in practice is that if the gain-bandwidth product of the op amp is not at least an order of magnitude greater than the cutoff frequency of the filter, there will be some interaction. If the gain-bandwidth product of the amplifier is greater than an order of magnitude higher than the filter, the response of the op amp can generally be ignored. In any case, if there is concern, the filter spice deck can be downloaded and the spice model of the specific op amp that will be used can be simulated with spice.

A current feedback amplifier can only be used with Sallen-Key topology since this is the only topology in which the op amp is configured as an amplifier. In the other topologies, the op amp is use with capacitors in the feedback network,

which is in appropriate for the current feedback amplifier.

Another choice is that of bipolar or FET input devices. In general, if the impedance level of the filter is less than 1 k Ω , a bipolar op amp is the appropriate choice. If the impedance is greater than 10 k Ω , a FET input op amp is a better choice. This is entirely due to the FET amps having a higher input impedance, which will be less of a load to the network.

A final word of caution, filters with high Q sections can cause the dynamic range of the op amp to be exceeded. This is due to peaking of the section. . The peaking due to Q will cause an amplitude, A_0 :

$$A_0 = H Q$$

where H is the gain of the circuit.

$$\alpha = \frac{1}{Q}$$

Also remember:

An Example

As an example, an antialiasing filter will now be designed.

The specifications for the filter are given as follows:

- 1) The cutoff frequency will be 8 kHz.
- 2) The stopband attenuation will be 72 dB. This corresponds to a 12 bit system.
- 3) Stopband frequency of 42 kSPS. This assumes a 100 KSPS A/D converter. The Nyquist frequency would be 50 kHz. Subtract 8 kHz., for the image of the passband around the sample rate gives us 42 kHz.
- 4) The Butterworth filter response is chosen in order to give the best compromise between attenuation and phase response.

Taking the Butterworth curves (Fig. 1) a line is drawn horizontally at 72 dB. A second line is drawn vertically at 5.25. This is the ratio of F_s/F_o . This is shown in Fig. 35.

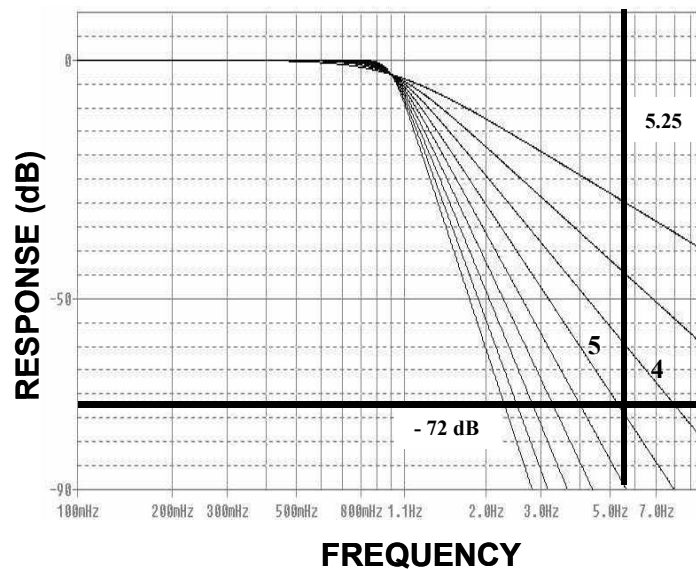


Figure 35: Determining Filter Order

This shows that a filter order of 5 is required.

This information is then used for input to the filter tool.

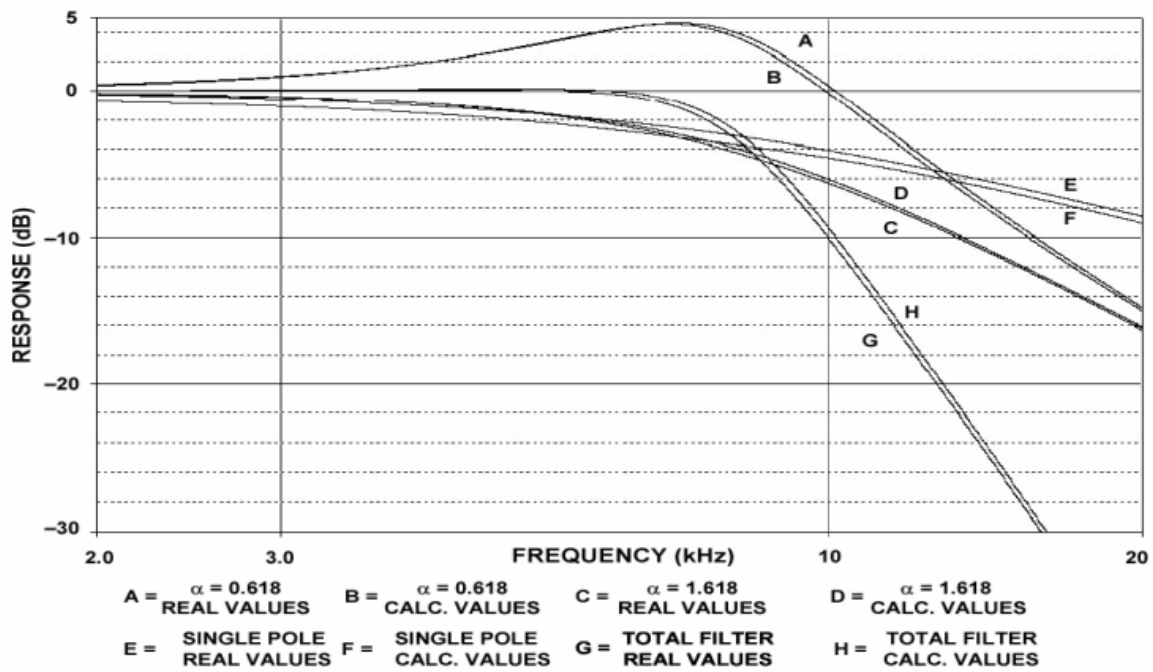


Fig. 36 Filter response change due to standard values

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